DIRECTORATE OF DISTANCE EDUCATION

UNIVERSITY OF NORTH BENGAL

MASTER OF ARTS- PHILOSOPHY

SEMESTER -IV

MODAL PROPOSITIONAL LOGIC (ELECTIVE)

ELECTIVE 404

BLOCK-1

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FOREWORD

The Self Learning Material (SLM) is written with the aim of providing simple and organized study content to all the learners. The SLMs are prepared on the framework of being mutually cohesive, internally consistent and structured as per the university's syllabi. It is a humble attempt to give glimpses of the various approaches and dimensions to the topic of study and to kindle the learner's interest to the subject

We have tried to put together information from various sources into this book that has been written in an engaging style with interesting and relevant examples. It introduces you to the insights of subject concepts and theories and presents them in a way that is easy to understand and comprehend.

We always believe in continuous improvement and would periodically update the content in the very interest of the learners. It may be added that despite enormous efforts and coordination, there is every possibility for some omission or inadequacy in few areas or topics, which would definitely be rectified in future.

We hope you enjoy learning from this book and the experience truly enrich your learning and help you to advance in your career and future endeavours.

MODAL PROPOSITIONAL LOGIC

BLOCK-1

BLOCK-2

BLOCK 1: MODAL PROPOSITIONAL LOGIC

Introduction to the Block

Unit 1 deals with Concept of Model Logic. Model theory began with the study of formal languages and their interpretations, and of the kinds of classification that a particular formal language can make.

Unit 2 deals with History of Model Logic. In this unit, an attempt is made to present a history of symbolic logic.

Unit 3 deals with Nature of Model Logic.It will introducing and familiarizing the definition, nature and scope of the subject expose the students to various definitions of logic.

Unit 4 deals with Logical interconnections between necessary, the impossible and permitted. These modal judgments and modal claims therefore play a central role in human decision-making and in philosophical argumentation. This entry is about the justification we have for modal judgments.

Unit 5 deals with Modal syllogisms. Syllogism is the most important part of Aristotle's logic. It is a kind of mediate inference in which conclusion follows from two premises.

Unit 6 deals with Stoic treatment of modality. Stoicism was one of the new philosophical movements of the Hellenistic period.

Unit 7 deals with Modal Logic. A modal is an expression (like 'necessarily' or 'possibly') that is used to qualify the truth of a judgement.

UNIT 1: CONCEPT OF MODEL LOGIC

STRUCTURE

1.0 OBJECTIVES

After this unit, we can able to know:

- To know about the Basic notions of model theory
- To discuss the Model-theoretic definition
- To know about Model-theoretic consequence
- To describe the Expressive strength
- To discuss about Models and modelling
- To understand the Set theory

1.1 INTRODUCTION

Model theory began with the study of formal languages and their interpretations, and of the kinds of classification that a particular formal language can make. Mainstream model theory is now a sophisticated branch of mathematics (see the entry on first-order model theory). But in a broader sense, model theory is the study of the interpretation of any language, formal or natural, by means of set-theoretic structures, with

Alfred Tarski's truth definition as a paradigm. In this broader sense, model theory meets philosophy at several points, for example in the theory of logical consequence and in the semantics of natural languages.

1.2 BASIC NOTIONS OF MODEL THEORY

Sometimes we write or speak a sentence S that expresses nothing either true or false, because some crucial information is missing about what the words mean. If we go on to add this information, so that S comes to express a true or false statement, we are said to interpret S, and the added information is called an interpretation of S. If the interpretation I happens to make S state something true, we say that I is a model of S, or that I satisfies S, in symbols 'I⊨S'. Another way of saying that I is a model of S is to say that S is true in I, and so we have the notion of model-theoretic truth, which is truth in a particular interpretation. But one should remember that the statement 'S is true in I' is just a paraphrase of 'S, when interpreted as in I, is true'; so model-theoretic truth is parasitic on plain ordinary truth, and we can always paraphrase it away.

For example I might say

He is killing all of them, and offer the interpretation that 'he' is Alfonso Arblaster of 35 The Crescent, Beetleford, and that 'them' are the pigeons in his loft. This interpretation explains (a) what objects some expressions refer to, and (b) what classes some quantifiers range over. (In this example there is one quantifier: 'all of them'). Interpretations that consist of items (a) and (b) appear very often in model theory, and they are known as structures. Particular kinds of model theory use particular kinds of structure; for example mathematical model theory tends to use so-called first-order structures, model theory of modal logics uses Kripke structures, and so on.

The structure I in the previous paragraph involves one fixed object and one fixed class. Since we described the structure today, the class is the class of pigeons in Alfonso's loft today, not those that will come tomorrow to replace them. If Alfonso Arblaster kills all the pigeons in his loft today, then I satisfies the quoted sentence today but won't satisfy it tomorrow, because Alfonso can't kill the same pigeons twice over. Depending on what you want to use model theory for, you may be happy to evaluate sentences today (the default time), or you may want to record how they are satisfied at one time and not at another. In the latter case you can relativise the notion of model and write '$I \vDash_t S$' to mean that I is a model of S at time t. The same applies to places, or to anything else that might be picked up by other implicit indexical features in the sentence. For example if you believe in possible worlds, you can index $\vDash$ by the possible world where the sentence is to be evaluated. Apart from using set theory, model theory is completely agnostic about what kinds of thing exist.

Note that the objects and classes in a structure carry labels that steer them to the right expressions in the sentence. These labels are an essential part of the structure.

If the same class is used to interpret all quantifiers, the class is called the domain or universe of the structure. But sometimes there are quantifiers ranging over different classes. For example if I say

One of those thingummy diseases is killing all the birds.

you will look for an interpretation that assigns a class of diseases to 'those thingummy diseases' and a class of birds to 'the birds'. Interpretations that give two or more classes for different quantifiers to range over are said to be many-sorted, and the classes are sometimes called the sorts.

The ideas above can still be useful if we start with a sentence S that does say something either true or false without needing further interpretation. (Model theorists say that such a sentence is fully interpreted.) For example we can consider misinterpretations I of a fully interpreted sentence S. A misinterpretation of S that makes it true is known as a nonstandard or unintended model of S. The branch of mathematics called nonstandard analysis is based on nonstandard models of mathematical statements about the real or complex number systems; see Section 4 below.

One also talks of model-theoretic semantics of natural languages, which is a way of describing the meanings of natural language sentences, not a way of giving them meanings. The connection between this semantics and model theory is a little indirect. It lies in Tarski's truth definition of 1933. See the entry on Tarski's truth definitions for more details.

1.3 MODEL-THEORETIC DEFINITION

A sentence S divides all its possible interpretations into two classes, those that are models of it and those that are not. In this way it defines a class, namely the class of all its models, written Mod(S). To take a legal example, the sentence

The first person has transferred the property to the second person, who thereby holds the property for the benefit of the third person.

defines a class of structures which take the form of labelled 4-tuples, as for example (writing the label on the left):

the first person = Alfonso Arblaster;

the property = the derelict land behind Alfonso's house;

the second person = John Doe;

the third person = Richard Roe.

This is a typical model-theoretic definition, defining a class of structures (in this case, the class known to the lawyers as trusts).

We can extend the idea of model-theoretic definition from a single sentence S to a set T of sentences; Mod(T) is the class of all interpretations that are simultaneously models of all the sentences in T. When a set T of sentences is used to define a class in this way, mathematicians say that T is a theory or a set of axioms, and that T axiomatises the class Mod(T).

Take for example the following set of first-order sentences:

$\forall x\forall y\forall z(x+(y+z)=(x+y)+z).\forall x(x+0=x).\forall x(x+(-x)=0).\forall x\forall y(x+y=y+x).$

Here the labels are the addition symbol '+', the minus symbol '−' and the constant symbol '0'. An interpretation also needs to specify a domain for the quantifiers. With one proviso, the models of this set of sentences are precisely the structures that mathematicians know as abelian groups. The proviso is that in an abelian group A, the domain should contain the interpretation of the symbol 0, and it should be closed under the interpretations of the symbols + and −. In mathematical model theory one builds this condition (or the corresponding conditions for other function and constant symbols) into the definition of a structure.

Each mathematical structure is tied to a particular first-order language. A structure contains interpretations of certain predicate, function and constant symbols; each predicate or function symbol has a fixed arity. The collection K of these symbols is called the signature of the structure. Symbols in the signature are often called nonlogical constants, and an older name for them is primitives. The first-order language of signature K is the first-order language built up using the symbols in K, together with the equality sign =, to build up its atomic formulas. (See the entry on classical logic.) If K is a signature, S is a sentence of the language of signature K and A is a structure whose signature is K, then because the symbols match up, we know that A makes S either true or false. So one defines the class of abelian groups to be the class of all those structures of signature +, −, 0 which are models of the sentences above. Apart from the fact that it uses a formal first-order language, this is exactly the algebraists' usual definition of the class of abelian groups; model theory formalises a kind of definition that is extremely common in mathematics.

Now the defining axioms for abelian groups have three kinds of symbol (apart from punctuation). First there is the logical symbol = with a fixed meaning. Second there are the nonlogical constants, which get their interpretation by being applied to a particular structure; one should group the quantifier symbols with them, because the structure also determines the domain over which the quantifiers range. And third there are the variables x,y etc. This three-level pattern of symbols allows us to define classes in a second way. Instead of looking for the interpretations of the nonlogical constants that will make a sentence true, we fix the interpretations of the nonlogical constants by choosing a particular structure A, and we look for assignments of elements of A to variables which will make a given formula true in A.

For example let Z be the additive group of integers. Its elements are the integers (positive, negative and 0), and the symbols +, −, 0 have their usual meanings. Consider the formula

v1+v1=v2.

If we assign the number −3 to v1 and the number −6 to v2, the formula works out as true in Z. We express this by saying that the pair (−3,−6) satisfies this formula in Z. Likewise (15,30) and (0,0) satisfy it, but (2,−4) and (3,3) don't. Thus the formula defines a binary relation on the integers, namely the set of pairs of integers that satisfy it. A relation defined in this way in a structure A is called a first-order definable relation in A. A useful generalisation is to allow the defining formula to use added names for some specific elements of A; these elements are called parameters and the relation is then definable with parameters.

This second type of definition, defining relations inside a structure rather than classes of structure, also formalises a common mathematical practice. But this time the practice belongs to geometry rather than to algebra. You may recognise the relation in the field of real numbers defined by the formula

v21+v22=1.

It's the circle of radius 1 around the origin in the real plane. Algebraic geometry is full of definitions of this kind.

During the 1940s it occurred to several people (chiefly Anatolii Mal'tsev in Russia, Alfred Tarski in the USA and Abraham Robinson in Britain) that the metatheorems of classical logic could be used to prove mathematical theorems about classes defined in the two ways we have just described. In 1950 both Robinson and Tarski were invited to address the International Congress of Mathematicians at Cambridge Mass. on this new discipline (which as yet had no name – Tarski proposed the name 'theory of models' in 1954). The conclusion of Robinson's address to that Congress is worth quoting:

[The] concrete examples produced in the present paper will have shown that contemporary symbolic logic can produce useful tools – though by no means omnipotent ones – for the development of actual mathematics, more particularly for the development of algebra and, it would appear, of algebraic geometry. This is the realisation of an ambition which was expressed by Leibniz in a letter to Huyghens as long ago as 1679.

In fact Mal'tsev had already made quite deep applications of model theory in group theory several years earlier, but under the political conditions of the time his work in Russia was not yet known in the West. By the end of the twentieth century, Robinson's hopes had been amply fulfilled; see the entry on first-order model theory.

There are at least two other kinds of definition in model theory besides these two above. The third is known as interpretation (a special case of the interpretations that we began with). Here we start with a structure A, and we build another structure B whose signature need not be related to that of A, by defining the domain X of B and all the labelled relations and functions of B to be the relations definable in A by certain formulas with parameters. A further refinement is to find a definable equivalence relation on X and take the domain of B to be not X itself but the set of equivalence classes of this relation. The structure B built in this way is said to be interpreted in the structure A.

A simple example, again from standard mathematics, is the interpretation of the group Z of integers in the structure N consisting of the natural numbers 0, 1, 2 etc. with labels for 0, 1 and +. To construct the domain of Z we first take the set X of all ordered pairs of natural numbers (clearly a definable relation in N), and on this set X we define the equivalence relation $\sim$ by

$(a,b)\sim(c,d)$ if and only if $a+d=b+c$

(again definable). The domain of Z consists of the equivalence classes of this relation. We define addition on Z by

$(a,b)+(c,d)=(e,f)$ if and only if $a+c+f=b+d+e$.

The equivalence class of (a,b) becomes the integer $a-b$.

When a structure B is interpreted in a structure A, every first-order statement about B can be translated back into a first-order statement about A, and in this way we can read off the complete theory of B from that of A. In fact if we carry out this construction not just for a single structure A but for a family of models of a theory T, always using the same defining formulas, then the resulting structures will all be models of a theory T′ that can be read off from T and the defining formulas. This gives a precise sense to the statement that the theory T′ is interpreted in the theory T. Philosophers of science have sometimes experimented with this notion of interpretation as a way of making precise what it means for one theory to be reducible to another. But realistic examples of reductions between scientific theories seem generally to be much subtler than this simple-minded model-theoretic idea will allow. See the entry on intertheory relations in physics.

The fourth kind of definability is a pair of notions, implicit definability and explicit definability of a particular relation in a theory. See section 3.3 of the entry on first-order model theory.

Unfortunately there used to be a very confused theory about model-theoretic axioms, that also went under the name of implicit definition. By the end of the nineteenth century, mathematical geometry had generally ceased to be a study of space, and it had become the study of classes of structures which satisfy certain 'geometric' axioms. Geometric terms like 'point', 'line' and 'between' survived, but only as the primitive symbols in axioms; they no longer had any meaning associated with them. So the old question, whether Euclid's parallel postulate (as a statement about space) was deducible from Euclid's other assumptions about space, was no longer interesting to geometers. Instead, geometers showed that if one wrote down an up-to-date version of Euclid's other assumptions, in the form of a theory T, then it was possible to find models of T which fail to satisfy the parallel postulate. (See the entry on geometry in the 19th century for the contributions of Lobachevski and Klein to this achievement.) In 1899 David Hilbert published a book in which he constructed such models, using exactly the method of interpretation that we have just described.

Problems arose because of the way that Hilbert and others described what they were doing. The history is complicated, but roughly the following happened. Around the middle of the nineteenth century people noticed, for example, that in an abelian group the minus function is definable in terms of 0 and + (namely: $-a$ is the element b such that $a+b=0$). Since this description of minus is in fact one of the axioms defining abelian groups, we can say (using a term taken from J. D. Gergonne, who should not be held responsible for the later use made of it) that the axioms for abelian groups implicitly define minus. In the jargon of the time, one said not that the axioms define the function minus, but that they define the concept minus. Now suppose we switch around and try to define plus in terms of minus and 0. This way round it can't be done, since one can have two abelian groups with the same 0 and minus but different plus functions. Rather than say this, the nineteenth century mathematicians concluded that the axioms only partially define plus in terms of minus and 0. Having swallowed that much, they went on to say that the axioms together form an implicit definition of the concepts plus, minus and 0 together, and that this implicit definition is only partial but it says about these concepts precisely as much as we need to know.

One wonders how it could happen that for fifty years nobody challenged this nonsense. In fact some people did challenge it, notably the geometer Moritz Pasch who in section 12 of his Vorlesungen über Neuere Geometrie (1882) insisted that geometric axioms tell us nothing whatever about the meanings of 'point', 'line' etc. Instead, he said, the axioms give us relations between the concepts. If one thinks of a structure as a kind of ordered n-tuple of sets etc., then a class Mod(T) becomes an n-ary relation, and Pasch's account agrees with ours. But he was unable to spell out the details, and there is some evidence that his contemporaries (and some more recent commentators) thought he was saying that the axioms may not determine the meanings of 'point' and 'line', but they do determine those of relational terms such as 'between' and 'incident with'! Frege's demolition of the implicit definition doctrine was masterly, but it came too late to save Hilbert from saying, at the beginning of his Grundlagen der Geometrie, that his axioms give 'the exact and mathematically adequate description' of the relations 'lie', 'between' and 'congruent'. Fortunately Hilbert's mathematics speaks for itself, and one can simply bypass these philosophical faux pas. The model-theoretic account that we now take as a correct description of this line of work seems to have surfaced first in the group around Giuseppe Peano in the 1890s, and it reached the English-speaking world through Bertrand Russell's Principles of Mathematics in 1903.

1.4 MODEL-THEORETIC CONSEQUENCE

Suppose LL is a language of signature K,TK,T is a set of sentences of LL and ϕϕ is a sentence of LL. Then the relation

Mod(T)⊆Mod(ϕ)Mod(T)⊆Mod(ϕ)

expresses that every structure of signature KK which is a model of TT is also a model of ϕϕ. This is known as the model-theoretic consequence relation, and it is written for short as

T⊨ϕT⊨ϕ

The double use of ⊨⊨ is a misfortune. But in the particular case where LL is first-order, the completeness theorem (see the entry on classical logic) tells us that 'T⊨ϕT⊨ϕ' holds if and only if there is a proof of ϕϕ from TT, a relation commonly written

T⊢ϕT⊢ϕ

Since ⊨⊨ and ⊢⊢ express exactly the same relation in this case, model theorists often avoid the double use of ⊨⊨ by using ⊢⊢ for model-theoretic consequence. But since what follows is not confined to first-order languages, safety suggests we stick with ⊨⊨ here.

Before the middle of the nineteenth century, textbooks of logic commonly taught the student how to check the validity of an argument (say in English) by showing that it has one of a number of standard forms, or by paraphrasing it into such a form. The standard forms were syntactic and/or semantic forms of argument in English. The process was hazardous: semantic forms are almost by definition not visible on the surface, and there is no purely syntactic form that guarantees validity of an argument. For this reason most of the old textbooks had a long section on 'fallacies' – ways in which an invalid argument may seem to be valid.

In 1847 George Boole changed this arrangement. For example, to validate the argument

All monarchs are human beings. No human beings are infallible. Therefore no infallible beings are monarchs.

Boole would interpret the symbols P,Q,RP,Q,R as names of classes:

PP is the class of all monarchs.

QQ is the class of all human beings.

RR is the class of all infallible beings.

Then he would point out that the original argument paraphrases into a set-theoretic consequence:

$(P\subseteq Q),(Q\cap R=0)\vDash(R\cap P=0)(P\subseteq Q),(Q\cap R=0)\vDash(R\cap P=0)$

(This example is from Stanley Jevons, 1869. Boole's own account is idiosyncratic, but I believe Jevons' example represents Boole's intentions accurately.) Today we would write $\forall x(Px\rightarrow Qx)\forall x(Px\rightarrow Qx)$ rather than $P\subseteq QP\subseteq Q$, but this is essentially the standard definition of $P\subseteq QP\subseteq Q$, so the difference between us and Boole is slight.

Insofar as they follow Boole, modern textbooks of logic establish that English arguments are valid by reducing them to model-theoretic consequences. Since the class of model-theoretic consequences, at least in first-order logic, has none of the vaguenesses of the old argument forms, textbooks of logic in this style have long since ceased to have a chapter on fallacies.

But there is one warning that survives from the old textbooks: If you formalise your argument in a way that is not a model-theoretic consequence, it doesn't mean the argument is not valid. It may only mean that you failed to analyse the concepts in the argument deeply enough before you formalised. The old textbooks used to discuss this in a ragbag section called 'topics' (i.e. hints for finding arguments that you might have missed). Here is an example from Peter of Spain's 13th century Summulae Logicales:

'There is a father. Therefore there is a child.' … Where does the validity of this argument come from? From the relation. The maxim is: When one of a correlated pair is posited, then so is the other.

Hilbert and Ackermann, possibly the textbook that did most to establish the modern style, discuss in their section III.3 a very similar example: 'If there is a son, then there is a father'. They point out that any attempt to justify this by using the symbolism

$\exists xSx\rightarrow\exists xFx\exists xSx\rightarrow\exists xFx$

is doomed to failure. "A proof of this statement is possible only if we analyze conceptually the meanings of the two predicates which occur", as they go on to illustrate. And of course the analysis finds precisely the relation that Peter of Spain referred to.

On the other hand if your English argument translates into an invalid model-theoretic consequence, a counterexample to the consequence may well give clues about how you can describe a situation that would make the premises of your argument true and the conclusion false. But this is not guaranteed.

One can raise a number of questions about whether the modern textbook procedure does really capture a sensible notion of logical consequence. For example in Boole's case the set-theoretic consequences that he relies on are all easily provable by formal proofs in first-order logic, not even using any set-theoretic axioms; and by the completeness theorem (see the entry on classical logic) the same is true for first-order logic. But for some other logics it is certainly not true. For instance the model-theoretic consequence relation for some logics of time presupposes some facts about the physical structure of time. Also, as Boole himself pointed out, his translation from an English argument to its set-theoretic form requires us to believe that for every property used in the argument, there is a corresponding class of all the things that have the property. This comes dangerously close to Frege's inconsistent comprehension axiom!

In 1936 Alfred Tarski proposed a definition of logical consequence for arguments in a fully interpreted formal language. His proposal was that an argument is valid if and only if: under any allowed reinterpretation of its nonlogical symbols, if the premises are true then so is the conclusion. Tarski assumed that the class of allowed reinterpretations could be read off from the semantics of the language, as set out in his truth definition. He left it undetermined what symbols count as nonlogical; in fact he hoped that this freedom would allow one to define different kinds of necessity, perhaps separating 'logical' from 'analytic'. One thing that makes Tarski's proposal difficult to evaluate is that he completely ignores the question we discussed above, of analysing the concepts to reach all the logical connections between them. The only plausible explanation I can see for this lies in his parenthetical remark about

the necessity of eliminating any defined signs which may possibly occur in the sentences concerned, i.e. of replacing them by primitive signs.

This suggests to me that he wants his primitive signs to be by stipulation unanalysable. But then by stipulation it will be purely accidental if his notion of logical consequence captures everything one would normally count as a logical consequence.

Historians note a resemblance between Tarski's proposal and one in section 147 of Bernard Bolzano's Wissenschaftslehre of 1837. Like Tarski, Bolzano defines the validity of a proposition in terms of the truth of a family of related propositions. Unlike Tarski, Bolzano makes his proposal for propositions in the vernacular, not for sentences of a formal language with a precisely defined semantics.

On all of this section, see also the entry on logical consequence.

1.5 EXPRESSIVE STRENGTH

A sentence SS defines its class Mod(S)Mod(S) of models. Given two languages LL and L′L′, we can compare them by asking whether every class Mod(S)Mod(S), with SS a sentence of LL, is also a class of the form Mod(S′)Mod(S′) where S′S′ is a sentence of L′L′. If the answer is Yes, we say that LL is reducible to L′L′, or that L′L′ is at least as expressive as LL.

For example if LL is a first-order language with identity, whose signature consists of 1-ary predicate symbols, and L′L′ is the language whose sentences consist of the four syllogistic forms (All AA are BB, Some AA are BB, No AA are BB, Some AA are not B)B) using the same predicate symbols, then L′L′ is reducible to LL, because the syllogistic forms are expressible in first-order logic. (There are some quarrels about which is the right way to express them; see the entry on the traditional square of opposition.) But the first-order language LL is certainly not reducible to the language L′L′ of syllogisms, since in LL we can write down a sentence saying that exactly three elements satisfy PxPx, and there is no way of saying this using just the syllogistic forms. Or moving the other way, if we form a third language L″L″ by adding to LL the quantifier QxQx with the meaning "There are uncountably many elements xx such that …", then trivially LL is reducible to L″L″, but the downward Loewenheim-Skolem theorem shows at once that L″L″ is not reducible to LL.

These notions are useful for analysing the strength of database query languages. We can think of the possible states of a database as structures, and a simple Yes/No query becomes a sentence that elicits the answer Yes if the database is a model of it and No otherwise. If one database query language is not reducible to another, then the first can express some query that can't be expressed in the second.

So we need techniques for comparing the expressive strengths of languages. One of the most powerful techniques available consists of the back-and-forth games of Ehrenfeucht and Fraïssé between the two players Spoiler and Duplicator; see the entry on logic and games for details. Imagine for example that we play the usual first-order back-and-forth game GG between two structures AA and BB. The theory of these games establishes that if some first-order sentence ϕϕ is true in exactly one of AA and BB, then there is a number nn, calculable from ϕϕ, with the property that Spoiler has a strategy for GG that will guarantee that he wins in at most nn steps. So conversely, to show that first-order logic can't distinguish between AA and BB, it suffices to show that for every finite nn, Duplicator has a strategy that will guarantee she doesn't lose GG in the first nn steps. If we succeed in showing this, it follows that any language which does distinguish between AA and BB is not reducible to the first-order language of the structures AA and BB.

These back-and-forth games are immensely flexible. For a start, they make just as much sense on finite structures as they do on infinite; many other techniques of classical model theory assume that the structures are infinite. They can also be adapted smoothly to many non-first-order languages.

In 1969 Per Lindström used back-and-forth games to give some abstract characterisations of first-order logic in terms of its expressive power. One of his theorems says that if LL is a language with a signature K,LK,L is closed under all the first-order syntactic operations, and LL obeys the downward Loewenheim-Skolem theorem for single sentences, and the compactness theorem, then LL is reducible to the first-order language of signature KK. These theorems are very attractive; see Chapter XII of Ebbinghaus, Flum and Thomas for a good account. But they have never quite lived up to their promise. It has been hard to find any similar characterisations of other logics. Even for first-order logic it is a little hard to see exactly what the characterisations tell us. But very roughly speaking, they tell us that first-order logic is the unique logic with two properties: (1) we can use it to express arbitrarily complicated things about finite patterns, and (2) it is hopeless for discriminating between one infinite cardinal and another.

These two properties (1) and (2) are just the properties of first-order logic that allowed Abraham Robinson to build his nonstandard analysis. The background is that Leibniz, when he invented differential and integral calculus, used infinitesimals, i.e. numbers that are greater than 0 and smaller than all of 1/2, 1/3, 1/4 etc. Unfortunately there are no such real numbers. During the nineteenth century all definitions and proofs in the Leibniz style were rewritten to talk of limits instead of infinitesimals. Now let RR be the structure consisting of the field of real numbers together with any structural features we care to give names to: certainly plus and times, maybe the ordering, the set of integers, the functions sin and log, etc. Let LL be the first-order language whose signature is that of RR. Because of the expressive strength of LL, we can write down any number of theorems of calculus as sentences of LL. Because of the expressive weakness of LL, there is no way that we can express in LL that RR has no infinitesimals. In fact Robinson used the compactness theorem to build a structure R′R′ that is a model of exactly the same sentences of LL as RR, but which has infinitesimals. As Robinson showed, we can copy Leibniz’s arguments using the infinitesimals in R′R′, and so prove that various theorems of calculus are true in R′R′. But these theorems are expressible in LL, so they must also be true in RR.

Since arguments using infinitesimals are usually easier to visualise than arguments using limits, nonstandard analysis is a helpful tool for mathematical analysts. Jacques Fleuriot in his Ph.D. thesis (2001) automated the proof theory of nonstandard analysis and used it to mechanise some of the proofs in Newton’s Principia.

1.6 MODELS AND MODELLING

To model a phenomenon is to construct a formal theory that describes and explains it. In a closely related sense, you model a system or structure that you plan to build, by writing a description of it. These are very different senses of ‘model’ from that in model theory: the ‘model’ of the phenomenon or the system is not a structure but a theory, often in a formal language. The Universal Modeling Language, UML for short, is a formal language designed for just this purpose. It's reported that the Australian Navy once hired a model theorist for a job 'modelling hydrodynamic phenomena'. (Please don't enlighten them!)

A little history will show how the word 'model' came to have these two different uses. In late Latin a 'modellus' was a measuring device, for example to measure water or milk. By the vagaries of language, the word generated three different words in English: mould, module, model. Often a device that measures out a quantity of a substance also imposes a form on the substance. We see this with a cheese mould, and also with the metal letters (called 'moduli' in the early 17th century) that carry ink to paper in printing. So 'model' comes to mean an object in hand that expresses the design of some other objects in the world: the artist's model carries the form that the artist depicts, and Christopher Wren's 'module' of St Paul's Cathedral serves to guide the builders.

Already by the late 17th century the word 'model' could mean an object that shows the form, not of real-world objects, but of mathematical constructs. Leibniz boasted that he didn't need models in order to do mathematics. Other mathematicians were happy to use plaster or metal models of interesting surfaces. The models of model theory first appeared as abstract versions of this kind of model, with theories in place of the defining equation of a surface. On the other hand one could stay with real-world objects but show their form through a theory rather than a physical copy in hand; 'modelling' is building such a theory.

We have a confusing halfway situation when a scientist describes a phenomenon in the world by an equation, for example a differential equation with exponential functions as solutions. Is the model the theory consisting of the equation, or are these exponential functions themselves models of the phenomenon? Examples of this kind, where theory and structures give essentially the same information, provide some support for Patrick Suppes' claim that "the meaning of the concept of model is the same in mathematics and the empirical sciences" (page 12 of his 1969 book cited below). Several philosophers of science have pursued the idea of using an informal version of model-theoretic models for scientific modelling. Sometimes the models are described as non-linguistic – this might be hard to reconcile with our definition of models in section 1 above.

Cognitive science is one area where the difference between models and modelling tends to become blurred. A central question of cognitive science is how we represent facts or possibilities in our minds. If one formalises these mental representations, they become something like ‘models of phenomena’. But it is a serious hypothesis that in fact our mental representations have a good deal in common with simple set-theoretic structures, so that they are ‘models’ in the model-theoretic sense too. In 1983 two influential works of cognitive science were published, both under the title Mental Models. The first, edited by Dedre Gentner and Albert Stevens, was about people’s ‘conceptualizations’ of the elementary facts of physics; it belongs squarely in the world of ‘modelling of phenomena’. The second, by Philip Johnson-Laird, is largely about reasoning, and makes several appeals to ‘model-theoretic semantics’ in our sense. Researchers in the Johnson-Laird tradition tend to refer to their approach as ‘model theory’, and to see it as allied in some sense to what we have called model theory.

Pictures and diagrams seem at first to hover in the middle ground between theories and models. In practice model theorists often draw themselves pictures of structures, and use the pictures to think about the structures. On the other hand pictures don’t generally carry the labelling that is an essential feature of model-theoretic structures. There is a fast growing body of work on reasoning with diagrams, and the overwhelming tendency of this work is to see pictures and diagrams as a form of language rather than as a form of structure. For example Eric Hammer and Norman Danner (in the book edited by Allwein and Barwise, see the Bibliography) describe a ‘model theory of Venn diagrams’; the Venn diagrams themselves are the syntax, and the model theory is a set-theoretical explanation of their meaning.

1.7 SET THEORY

Set theory (which is expressed in a countable language), if it is consistent, has a countable model; this is known as Skolem's paradox, since there are sentences in set theory which postulate the existence of uncountable sets and yet these sentences are true in our countable model. Particularly the proof of the independence of the continuum hypothesis requires considering sets in models which appear to be uncountable when viewed from within the model, but are countable to someone outside the model.

The model-theoretic viewpoint has been useful in set theory; for example in Kurt Gödel's work on the constructible universe, which, along with the method of forcing developed by Paul Cohen can be shown to prove the (again philosophically interesting) independence of the axiom of choice and the continuum hypothesis from the other axioms of set theory.

In the other direction, model theory itself can be formalized within ZFC set theory. The development of the fundamentals of model theory (such as the compactness theorem) rely on the axiom of choice, or more exactly the Boolean prime ideal theorem. Other results in model theory depend on set-theoretic axioms beyond the standard ZFC framework. For example, if the Continuum Hypothesis holds then every countable model has an ultrapower which is saturated (in its own cardinality). Similarly, if the Generalized Continuum Hypothesis holds then every model has a saturated elementary extension. Neither of these results are provable in ZFC alone. Finally, some questions arising from model theory (such as compactness for infinitary logics) have been shown to be equivalent to large cardinal axioms.

Other basic notions

Reducts and expansions

A field or a vector space can be regarded as a (commutative) group by simply ignoring some of its structure. The corresponding notion in model theory is that of a **reduct** of a structure to a subset of the original signature. The opposite relation is called an expansion - e.g. the (additive) group of the rational numbers, regarded as a structure in the signature {+,0} can be expanded to a field with the signature {×,+,1,0} or to an ordered group with the signature {+,0,<}.

Similarly, if σ' is a signature that extends another signature σ, then a complete σ'-theory can be restricted to σ by intersecting the set of its sentences with the set of σ-formulas. Conversely, a complete σ-theory can be regarded as a σ'-theory, and one can extend it (in more than one way) to a complete σ'-theory. The terms reduct and expansion are sometimes applied to this relation as well.

Interpretability

Given a mathematical structure, there are very often associated structures which can be constructed as a quotient of part of the original structure via an equivalence relation. An important example is a quotient group of a group.

One might say that to understand the full structure one must understand these quotients. When the equivalence relation is definable, we can give the previous sentence a precise meaning. We say that these structures are **interpretable**.

A key fact is that one can translate sentences from the language of the interpreted structures to the language of the original structure. Thus one can show that if a structure M interprets another whose theory is undecidable, then M itself is undecidable.

Using the compactness and completeness theorems

Gödel's completeness theorem (not to be confused with his incompleteness theorems) says that a theory has a model if and only if it is consistent, i.e. no contradiction is proved by the theory. This is the heart of model theory as it lets us answer questions about theories by looking at models and vice versa. One should not confuse the completeness theorem with the notion of a complete theory. A complete theory is a theory that contains every sentence or its negation. Importantly, one can find a complete consistent theory extending any consistent theory. However, as shown by Gödel's incompleteness theorems only in relatively simple cases will it be possible to have a complete consistent theory that is also recursive, i.e. that can be described by a recursively enumerable set of axioms. In particular, the theory of natural numbers has no recursive complete and consistent theory. Non-recursive theories are of little practical use, since it is undecidable if a proposed axiom is indeed an axiom, making proof-checking a supertask.

The compactness theorem states that a set of sentences S is satisfiable if every finite subset of S is satisfiable. In the context of proof theory the analogous statement is trivial, since every proof can have only a finite number of antecedents used in the proof. In the context of model theory, however, this proof is somewhat more difficult. There are two well known proofs, one by Gödel (which goes via proofs) and one by Malcev (which is more direct and allows us to restrict the cardinality of the resulting model).

Model theory is usually concerned with first-order logic, and many important results (such as the completeness and compactness theorems) fail in second-order logic or other alternatives. In first-order logic all infinite cardinals look the same to a language which is countable. This is expressed in the Löwenheim–Skolem theorems, which state that any countable theory with an infinite model has models of all infinite cardinalities (at least that of the language) which agree with on all sentences, i.e. they are 'elementarily equivalent'. Many important properties in model theory can be expressed with types. Further many proofs go via constructing models with elements that contain elements with certain types and then using these elements.

History

Model theory as a subject has existed since approximately the middle of the 20th century. However some earlier research, especially in mathematical logic, is often regarded as being of a model-theoretical nature in retrospect. The first significant result in what is now model theory was a special case of the downward Löwenheim–Skolem theorem, published by Leopold Löwenheim in 1915. The compactness theorem was implicit in work by Thoralf Skolem, but it was first published in 1930, as a lemma in Kurt Gödel's proof of his completeness theorem. The Löwenheim–Skolem theorem and the compactness theorem received their respective general forms in 1936 and 1941 from Anatoly Maltsev.

The development of model theory can be traced to Alfred Tarski, a member of the Lwów–Warsaw school during the interbellum. Tarski's work included logical consequence, deductive systems, the algebra of logic, the theory of definability, and the semantic definition of truth, among other topics. His semantic methods culminated in the model theory he and a number of his Berkeley students developed in the 1950s and '60s. These modern concepts of model theory influenced Hilbert's program and modern mathematics.

Check Your Progress 1

Notes: a) Space is given below for your answers.

b) Compare your answer with the one given at the end of this unit.

1. How do you know about the Basic notions of model theory?

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2. Discuss the Model-theoretic definition.

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3. What do you know about Model-theoretic consequence?

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4. Describe the Expressive strength.

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1.8 LET US SUM UP

The model theorist Yuri Gurevich introduced abstract state machines (ASMs) as a way of using model-theoretic ideas for specification in computer science. According to the Abstract State Machine website (see Other Internet Resources below), any algorithm can be modeled at its natural abstraction level by an appropriate ASM. … ASMs use classical mathematical structures to describe states of a computation; structures are well-understood, precise models.

The book of Börger and Stärk cited below is an authoritative account of ASMs and their uses.

Today you can make your name and fortune by finding a good representation system. There is no reason to expect that every such system will fit neatly into the syntax/semantics framework of model theory, but it will be surprising if model-theoretic ideas don't continue to make a major contribution in this area.

1.9 KEY WORDS

Set Theory: Set theory is a branch of mathematical logic that studies sets, which informally are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics.

Model: Logic models are hypothesized descriptions of the chain of causes and effects leading to an outcome of interest.

1.10 QUESTIONS FOR REVIEW

1. To discuss about Models and modelling
2. To understand the Set theory

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1.12 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 1.2
2. See Section 1.3
3. See Section 1.4
4. See Section 1.5

UNIT 2: HISTORY OF MODEL LOGIC

STRUCTURE

2.0 OBJECTIVES

In this unit, an attempt is made to present a history of symbolic logic. You will be quick enough: to notice that the moment you enter symbolic logic, you are confronted with mathematics as well.

- to learn that development of logic and mathematics are inseparably related.
- to know that logic and mathematics are two components of one enterprise.
- to be familiar with conceptual developments with a brief description of what they are.
- to set your priorities right, to identify the elements of logic in mathematical discussions.

2.1 INTRODUCTION

History of logic can safely be divided into three phases; ancient logic, medieval logic and modern logic. It is necessary to bear in mind that one is not just replacement for the other and that elements of later phase can be discerned in the earlier phase. Therefore development is significantly in terms of correction and improvements, but not total rejection.

Therefore it is absolutely necessary to admit that the limitations of ancient and medieval systems of logic paved way for the rise of symbolic logic and its value in addition to pioneering work by some mathematicians.

2.2 EARLIEST CONTRIBUTIONS TO LOGIC

The greatest contribution of Aristotle to logic, undoubtedly, is his theory of syllogism in which the theory of classes and class relation is implicit. Another significant contribution of Aristotle is his notion of variables. Classes themselves are variables in the sense that in any proposition subject and predicate terms are not only variables but also they are the symbols of classes.

Finally, the class relation, which is explicit in his four-fold analysis of categorical proposition, is understood as inclusion or exclusion - total or partial. Theophrastus, a student of Aristotle, developed a theory of pure hypothetical syllogism. A hypothetical syllogism is said to be pure if all the three propositions are hypothetical propositions. Theophrastus showed that pure hypothetical inference (an inference which consists of only hypothetical propositions) could be constructed which corresponds to inference consisting of only categorical propositions (which Aristotle called syllogism). A school of thought flourished during Socrates' period known as Megarians.

The first generation of Megarians flourished in the 5th century B.C. onwards. In the 4th century B.C. one Megarian by name Eubulides of Miletus introduced now famous paradox – the paradox of liar. The last Greek logician, (who is also 'lost' because none of his writings is extant), who is worthy of consideration is Chrysippus of whom it is said that even gods would have used the logic of Chrysippus if they had to use logic. Peter Abelard, who lived in the 11th Century, is generally regarded as the first important logician of medieval age followed by William of Sherwood and Peter of Spain in the 13th Century. They continued the work of Aristotle on categorical proposition and syllogism and other related topics. In reality, no vacuum was created in medieval age and hence there was continuity from Aristotelian logic to modern logic though no original contribution came from any logician. The most notable contribution to logic in this period consists in the developments, which took place in several important fields like analysis of syntax and semantics of natural language, theories of reference and application, philosophy of language, etc., the relevance of which was, perhaps realized only very recently. These are precisely some of the topics of modern logic.

William of Sherwood and Peter of Spain were the first to make the distinction between descriptive and nondescriptive functions of language. They reserved the word 'term' only for descriptive function. Accordingly, only subject and predicate qualify for descriptive function and hence in categorical proposition we can find only two terms. These were called categorematic whereas other components of a sentence like 'all, some, and no', etc. were called syncategorematic. The former are terms whereas the latter are only words. Hence, terms were regarded as special words. It is in this context that the medieval logicians made semantic distinction of language levels. Categorematic term was divided into two classes, terms of first intension and terms of second intension. First class stands for things whereas the second stands not for a thing but for a language sign. In a limited sense, and at elementary level, it can be said that subject represents first class and predicate represents second class. Another field covered by medieval logicians was that of quantification which is of great importance in modern logic. again, this is another important topic of modern logic.

2.3 LIMITATIONS OF ARISTOTELIAN LOGIC

The very fact that Aristotle constructed an extraordinarily sound system of logic became its nemesis. Just as Newtonian Physics was held as infallible for a little more than two hundred years, Aristotle was held on similar lines for nearly two thousand years. However, neither of them anticipated this treatment to their systems. While this is one reason for the delayed beginning of modern logic, second and the most important reason is that mathematics also had not yet been developed. The emphasis is not upon the defects of the system, but on the limitations because, ironically, the defects did not hinder the growth of logic. It may also be true that had the defects been detected very early, situation would not have been much different because time was not ripe for take-off of symbolic logic. One serious limitation of Aristotelian system is its narrow conception of proposition. He restricted it to subject-predicate form. Though class-relation is implicit in this theory of syllogism, Aristotle ignored it. There is little wonder that Aristotle did not think of any other relations. Consider these two examples:

All men are mortal.
All mortal beings are imperfect

All men are imperfect.
Bangalore is to the east of Mangalore.
Madras is to the east of Bangalore.

Madras is to the east of Mangalore.

Both these arguments are valid in virtue of transitive relation. Aristotle recognized only the first example as valid and what is surprising is that he considered only the first type as an argument. The result is that most of the mathematical statements ceased to be propositions in his analysis. His narrow outlook eliminated any possibility of logic and mathematics interacting. Consequently, considerable types of arguments with much complicated structure fall outside the limits of Aristotelian logic and hence remain unexamined. Medieval logic, in spite of remarkable contributions to logic, did not take logic a step ahead because whatever research was done was only an in-house work, i.e., work within the system. What was required was transition from one system to another.

In what sense modern logic makes progress over Aristotelian logic? It is very important to answer this question. Modern logic did not supersede Aristotelian logic in the sense in which an amendment to constitution results in one act replacing another. Modern logic neither superseded nor succeeded Aristotelian logic. It only extended the boundaries of the system. Existing rules remained not only acceptable but also were augmented by new set of rules. Later we will learn that among nine rules of inference, six are from Aristotelian logic. And simple conversion and observation were retained but given 'extended meaning' in terms of the rules of commutation and double negation respectively. Meaning was extended because logic and mathematics mutually made inroads into one another's territory. In a similar fashion, the use of variables also underwent a change. While Aristotle used variables only to represent terms, modern logic extended the use to propositions as well. This inclusion had far reaching consequences. Lastly, quantification, which was introduced during medieval age, was further improvised.

The foregoing discussion should make one point clear. The tools used to test arguments or to construct arguments by Aristotelian system are insufficient. Modern logic further augmented the tools not only in number but also in variety. It should be remembered that the sky is the limit to improve and add. Before we enter the modern era, one interesting question must be considered. How should we explain the relation between logic and mathematics? Two philosophers have differently described this relation. Raymond Wilder says that for Peano and his followers 'logic was the servant of mathematics'. Wilder put it in a more respectable and acceptable form, in connection with Frege's philosophy of mathematics, 'dependence (of mathematics) on logic… was more like that of child to parent than servant to master. Basson and O'connor have echoed more or less similar views while relating classical logic to modern logic. It is like embryo related to adult.

2.4 HISTORY AND UTILITY OF SYMBOLIC LOGIC

At this stage, two aspects must be made clear. Modern logic is also called symbolic logic because symbols replaced words to a great extent. Second, symbolic logic and mathematics do not stand sundered; so much so, modern logic is also called mathematical logic, which A.N. Prior terms 'loosely called.' However, Prior's remark has to be taken with a pinch of salt. Very soon, we realize that almost all people, whose names are associated with symbolic logic, are basically mathematicians. And at some stage it becomes extremely difficult to separate logic from mathematics and, if attempted, it will be an exercise in futility. However, a definite limitation must be considered. When we talk of mathematics we talk of pure mathematics only. So when we deal with history of a symbolic logic we deal with the history of pure mathematics. Where exactly does symbolic logic score over classical logic? Language is, generally, ambiguous. It is so for two reasons. In the first place, a significant number of words are equivocal and secondly, many times the construction of sentences and their juxtaposition are misleading so much so they convey meaning very different from what the speaker or author intends. Replacement of words by symbols and application of logical syntax different from grammatical syntax completely eliminates ambiguity. The meaning of logical syntax becomes clear in due course when sentences are represented by symbols. It is possible to test the validity of arguments only when the statements are unambiguous. Further, use of symbols saves time and effort required to test the validity of arguments.

2.5 THE RISE OF SYMBOLIC LOGIC

Generally, bibliography of symbolic logic compiled by Alonzo Church is reckoned as authentic to determine the beginning of symbolic logic. In the year 1666, Leibniz published (or wrote) a thesis on a 'Theory of Combinations titled 'Dissertatio de Arte Combinatoria.' It is said that the beginning of symbolic logic coincides with this work. If so, Chrysippus has to be heralded as the forerunner of symbolic logic because according to records long before Leibniz he showed some interest in Combinations. So he must have done some work on Combinations, which was, further, followed up by some logicians in the thirteenth century. In brief, let us describe the subject-matter of Combinations. Leibniz was more concerned with such issues as semantic interpretations of logical formulas. One example may clarify semantic consideration or considerations which engaged Leibniz. What does the statement 'All men are mortal' mean? Does it mean that every member of the class of men is also a member of the class of mortal beings? Or does it mean that every man possesses the attribute of being a mortal? Or does it mean that the attribute of 'being man' includes the 'attribute of being mortal'. In other words, the focus of this consideration is on the choice between extensional approach and intentional approach. Class-membership issue is extensional whereas attribute-inclusion or attribute – exclusion is intentional. Another notable contribution of Leibniz was his work on logical algebra or logical calculus, which consists of several experimental sorts of studies. Some laws, which are features of his study, are laws of identity and explicit statement of transitive relation, which made Aristotelian syllogism significant. Consider these two rules:

a b is a

ab is b

These rules become intelligible when we substitute terms for a & b. suppose that a = intelligent; b = man 1) Intelligent man is a man 2) Intelligent man is intelligent

Likewise consider another rule:

if a is b and a is c then a is bc.

Again substitute of, b and c, a = Indian, b = Asian, c = Hindu. Then 3 becomes If Indian is an Asian and Indian is a Hindu, then Indian is an Asian Hindu.

An important requirement of logical algebra is that substitution must be possible; this particular relation was explicitly recognized by Leibniz. In the 18th century two mathematician, Euler and Lambert contributed to the development of logic. While Euler is known for geometrical representation of propositions through his circles, Lambert developed logical calculus on intensional lines. For example, if a and b are two concepts, then a + b becomes a complex concept and ab stands for conceptual element common to a and b. What applies to class membership applies also to attributes. Bolzano is another logician who contributed to logic in the 19th century. He regarded terms and propositions as fundamental constituents of logic. He is known for an extraordinary approach to the logical semantics of language. In this context, he regarded propositions as having universal application when certain conditions are satisfied and as universally inapplicable under certain other conditions and as consistent under certain other conditions. Bolzano in fact, modified Kant's definition of 'analytic judgment' using this particular criterion. Another important contribution of Bolzano was his conception of probability. He introduced some modifications into Laplace's conception of probability, which was widely held during his time. Laplace defined probability as equipossible while determining the probability value when only two possibilities are available as in the case of tossing of the coin. In fact, Bolzano's modification avoids this particular element. This is crucial because 'equipossible' involves circularity. By avoiding this term, Bolzano could avoid circularity, which was inherent in Laplace's theory.

In 1847 two mathematicians, de Morgan and George Boole published 'Formal Logic' and 'The Mathematical Analysis of Logic respectively. Symbolic logic actually took off from this point of time. De Morgan gave to the world of logic now famous notion of complement which was later exploited by John Venn to geometrically represent distribution of terms and test syllogistic arguments. De Morgan showed that if there are two classes, then there are four product classes and Jevons showed that if there are three classes, then there are eight product classes. So generalizing this relation, we can say that the relation between the number of classes and the number of product classes is given by the formula, n = 2x . Where, 'n' stands for the number of product-classes and x stands for the number of terms. This formula is only indicative of the type of relation, which holds good between classes (or sets) and product classes because there is no syllogism with more than three terms and no proposition (in traditional sense) has more than two terms. He also gave a formula known as de Morgan's law to write the contradiction for disjunctive and conjunctive propositions. Boole's contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work 'An Investigation of the Laws of Thought!' It is in this work that the germs of the 20th century symbolic logic can be traced. While Lambert invented union of concepts on intensional analysis. Boole invented union of sets on extensional basis. He used '1' to designate the universe. Following de

Morgan, Boole called it the universe of discourse. He introduced the following laws, which play crucial role in mathematical logic.

1 Union of any set and universal set is a universal set. Let X be a set. Then 1+X = 1

2 Product of a universal set and any non-null set X is X itself.

3 Product of null-set and any non-null set (universal set included) is a null-set itself.

If X is a non-null set, the 1 – X is its complementary.

5 It is self-evident that product of any non-null set and its complementary is a null-set.

5 Stands for Boole's definition of contradiction. He also showed that if X, Y, Z,…etc. stand for non- null sets, then all laws of algebra hold good. Most important among them are what are known as distributive and commutative laws. For the sake of brevity, these laws are stated as follows:

1 Distributive Law: a(b+c) = ab + ac

2 Commutative Law: ab= ba

or a+b=b+a

Using the concept of complementary class, Boole also showed that 'A, E, I and O' of traditional logic can be reinterpreted. His suggestion was geometrically represented by Venn. In this interpretation, Boole actually considered what is called class logic, which later became the cornerstone of set theory. In logic, there is another topic called calculus of propositions. Boole integrated these two and defined the truth-value of what are called compound propositions which also consist of variables. While in the first interpretation the variables represent the sets or terms, in the second interpretation they represent the propositions. Consequently, products of classes, here, become conjunction and union or addition of classes becomes disjunction. Complement of a set becomes negation of a proposition.

Boolean analysis of logic is also called Boolean algebra for two reasons. In the first place, he freely used variables to explain various aspects of logic. Extensive use of variables characterizes algebra. Secondly, he defined all four operations of algebra; addition, multiplication, subtraction and division and extended the same to logic. Venn's contribution to logic was partially mentioned earlier. Therefore the remaining part requires to be mentioned. Venn is well-known for making qualitative distinction, in addition to traditionally held quantitative distinction between universal and existential (particular) which has far reaching consequences. The distinction is that while universal proposition (in modern logic universal quantifier) denies the existence of membership in a class, existential quantifier affirms the same. Secondly, a large number of deductive inferences became invalid as a result of this description. The irony is that in this situation, progress is marked not by augmentation but by depletion in the number of inferences. There were certain anomalies in Boolean system. Consider two identical sets, say X and Y where every member of X is a member of Y and every member of Y is a member of X; for example, the class of bachelors and the class of unmarried men. The product class should yield X Y. Since Y = X, XY=X2 or Y2 . In algebra it makes sense, but surely not in logic. Similarly X+Y, the union of two sets ought to become 2X. Again, it holds good in algebra but not in logic. Jevons, a student of de Morgan, succeeded in eliminating these anomalies; according to his interpretation, the union of two identical sets does not double the strength, say from n to 2n. The reason is simple; every member is present in both the sets. We cannot count one individual as two just because he or it is present in two sets simultaneously. The same reasoning applies to product of identical sets. If there are 100 bachelors and 100 unmarried men then the product of these two sets does not produce 1002 = 10,000 bachelors who are also unmarried men, but 100 only. C.S. Peirce resolved this anomaly in a different way. He identified logical addition with inclusive or instead of exclusive or (either p or q but not both is an example for exclusive or and either p or q or both is an example for inclusive or).

Peirce introduced a symbol ⊃ for class inclusion. He strangely argued that there is no difference between a proposition and inference or implication. In the ultimate analysis only implication survives. Secondly, all implications have quantifiers, which may be explicit or implicit. While Peirce thought that implication is the primary constituent of logic, at a later stage, there were attempts to eliminate implication and retain only negation and conjunction. While introducing symbols in a set of formulas Peirce was driven by a definite motive. He believed that symbols should resemble what they represent say thoughts. To achieve his aim, Peirce used, what he called, 'existential graphs'. They were not graphs in geometrical sense. He regarded parentheses themselves as graphs. For example, 'if p, then q'was represented graphically, by Peirce by using parentheses. He inserted p and q within parentheses and represented as (p (q)).

Christine Ladd Franklin invented a new technique of testing syllogism called antilogism or inconsistent triad. In addition to, Venn's diagram, antilogism also eliminated weakened and strengthened moods on the ground that particular propositions cannot be deduced from universal propositions only.

Gottlob Frege is one of the pioneers, who gave a new dimension to mathematical logic. In 1879 'Begriffsschrift' the first of his most important works was published followed by Die Grudlagan der Arithamatik in 1884. His first work dealt with proper symbolization with the help of rules of quantification. His intention was to codify logical principles used in mathematical reasoning like substitution, modus ponens, etc. In this work he introduced the notion of function, which was later renamed as propositional function. He also introduced a system of basic formulas for propositions in terms of implication and negation. In his second work, Frege made the most crucial attempt to trace the roots of mathematics to logic. He himself regarded arithmetic as simply a development of logic. Consequently, every proposition of arithmetic became merely a law of logic. History has recorded that Frege's thesis would not have got what it deserved but for Russell's discovery of Frege. Hence the relation between arithmetic and logic is known as Frege-Russell thesis. It is said that modern logic began with Frege. It means that in one sense the history of symbolic logic stops before Frege. Whatever development that took place after Frege's period characterize contemporary logic. Even in this period, there were remarkable changes with new theses being presented regularly. Giuseppe Peano tried to establish the relation between logic and mathematics in a slightly different manner. Instead of tracing the roots of mathematics to logic, Peano tried to express mathematical methods in a different form similar to that of logical calculus. For example, the successor of 'a' was designated by the symbol 'a+'; also in addition to the symbol ⊃ he introduced another symbol ∈. This shows that implication or class inclusion (⊃) is distinct from 'element of' or 'belongs to'. In Peano's system there is no interpretation of any symbol and hence mathematics becomes a formal system. In the beginning of the 20th century Zermelo proposed his theory of sets known as Axiomatic Set theory. He intended his theory to be free from contradictions. He regarded it as well ordered because it was axiomatized. His claim was totally rejected by Poincare. Perhaps only two mathematicians disputed the theory that mathematics has its foundations in logic. Opposition to this approach developed first in the 19th century. Kronecker, a professor of mathematics at the University of Berlin in 1850s, was the first mathematician to oppose this dominant trend. He disagreed with Cantor's theory of sets which included the concept of infinity. Kronecker went to the extent of arguing that integers are made by God, but everything else is the work of man. After Kronecker, it was Poincare who believed that mathematics does not have its base in logic. His main thesis is that in the first place, mathematical induction cannot be reduced to logic; secondly, according to him, even mathematics proceeds from particular to universal only; a clear opposition to deductive logic.

2.6 THE AGE OF PRINCIPIA MATHEMATICA (PM)

In 1910 Bertrand Russell in association with A.N. Whitehead published Principia Mathematica. What was referred to as the Frege-Russel thesis in the previous section found exposition in this work. Only a few aspects of this great work can be dealt here. The principal thesis remains the same, that mathematics is an extension of logic. Jevons, earlier, remarked that 'algebra' is nothing but highly developed logic' to which Frege added: ‘inferences….. are based on general laws of logic.’ Frege was actually referring to mathematical induction. In the preface itself the authors admitted that ‘thanks to Peano and his followers symbolic logic… acquired the technical and the logical comprehensiveness that are essential to a mathematical instrument’. Clearly, the new age mathematicians bypass Poincare and Kronecker in this regard. PM makes a clear distinction between proposition and propositional function. While variables constitute propositional function, substitutions to variables constitute propositions. The former is neither true nor false. But the latter is either true or false. For example, X is the husband of Y is neither true nor false. But Rama (X) is the husband of Sita (Y) is true. A key logical term, which finds place in PM is material implication. Russell and Whitehead used ‘⊃’ to designate implication. Material implication is defined as follows: p⊃q ≡ ~ p q ˅ Truth-values were assigned by PM as follows. Both p and q can be true together, or when p is false, q may be false or true. But when p is true q cannot be false. Implication, therefore, does not imply necessary connection. To distinguish implication from prohibited possibility Russell and Whitehead used material implication instead of mere ‘implication’. This particular definition of material implication has a very important consequence. ‘Necessary relation’ was an unwanted metaphysical baggage, which was overthrown by Hume. But there was no way of interpreting implication in the absence of necessary relation. Fixation of truthvalue by PM made a distinct advance in this case. And it is precisely this type of implication that is used in mathematics. Consider a very familiar example, ‘If ABC is a plane triangle, then the sum of the three angles equals two right angles’. That there is no plane triangle at all does not affect the relation because even when the antecedent is false the consequent can continue to be true. Hence it comes to mean that a true premise can imply only true conclusion whereas a false premise can imply either true or false conclusion. PM includes five axioms (Russell and Whitehead use the word ‘principle’), which can be regarded as primitive logical truths. They are follows:

regarded as primitive logical truths. They are follows:

1 Tautology (Taut)

2 Addition (Add)

3 Permutation (Perm)

4 Association (Assoc)

5 Summation (Sum)

Example provided here is taken from the text itself. The authors in all these cases use the

Symbol I- which is read 'it is asserted that' or it is true that' and the dots after assertion I– sign

Indicate range. 'v' is read 'or' and '⊃ is read 'if...then'.

Taut: I-: p p. ˅ ⊃ .P It is true that p or p implies p.

Add: I-: q. ⊃ . p q It is true that if q, then p or q. ˅

Perm: I-: p q. ˅ ⊃ .q p If p or q, then q or p. ˅

Assoc: I-: p ˅ (q ˅ r). ⊃. q ˅ (p˅ r) If p or q or r, then q or p or r.

Sum: I- :q ⊃ r. ⊃: p q. ˅ ⊃ .p r If q implies r, then p or q implies p or r.

For 'Add' the example is 'if today is Wednesday (q), then today is either Tuesday or Wednesday. The examples can be constructed on similar lines for other axioms. For perm, the example read as follows; if today is Wednesday or Tuesday, then today is Tuesday or Wednesday. In all cases, the sentences are preceded by 'it is true that'. The colleen immediately after the assertion sign indicates range, but the dots which follow or precede variables are only customary. PM also includes equivalence relation, which explains the equivalence of the law of the Excluded Middle and the Law of contradiction. In the beginning of the summary of *3 the authors say that 'it is false that either p is false or q is false, which is obviously true when and only when p and q are both true. Symbolically, p.q = . ~ (~ p˅ ~ q) Reductio ad absurdum is one method accepted by mathematics. It means that the contradiction of what has to be proved is assumed to be true and then the conclusion contradicting the assumption is deduced. This contradiction shows that the assumption is false in which case its contradiction must be true. This is again a primitive logical truth. The principle of double negative is another, which can be easily derived from the law of the Excluded Middle. David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore, a consistent system, in Hilbert's analysis is an axiomatized system. A distinguishing mark of Hilbert's analysis is his 'discovery' of 'ideal limit'. From the days of Cantor and Weirstrass who introduced the concept of 'infinity' or 'transfinite' the concept of ideal limit engaged the attention of mathematicians. While elementary number theory could be empirically interpreted, infinity could not be interpreted in that manner. So Hilbert chose to regard transfinite as limit. There should not be break in history – circuit. Therefore another contribution of Hilbert secures a place in our discussion. Hilbert embarked upon his project to defend classical mathematics from one theory of mathematics known as intuitionism spearheaded by the Dutch mathematician Jan Brouwer, according to whom mathematics is not a system of formulas but is a sort of abstract activity, which abstracts the concept of 'numberness.' By any standard, 'intuitionist mathematics ceases to be a logical enterprise, but confines itself to the narrow domains of psychological activity at best and some sort of esoteric activity at worst. Following the tradition of PM, Emil Post presented the method of truth-tables published as 'Introduction to a General Theory of Propositions' in the American Journal of Mathematics in 1921. In this paper, Post included not only classical logic, which allowed only two values but a system allowing many values. In the same year Wittgenstein's Tractus logico-Philosophicus was published, which also included this technique. Wittgenstein held the view that mathematics is nothing but a bundle of tautologies. While this is the view of earlier Wittgenstein, in later Wittgenstein the conception of mathematics underwent dramatic change. In 'Remarks on the Foundations of

Mathematics' Wittgenstein argues that both logic and mathematics form parts of language games. At this point of time he became a conventionalist and argued that mathematical propositions are immune to falsification. This position of Wittgenstein is much closer to intuitionism than to anything else. Rudolph Carnap's contribution to symbolic logic consists in the extension of the same to epistemology and philosophy of science. He argued that all meaningful sentences belong to the language of science. He followed what is called the 'principle of tolerance' with which any form of expression could be defended if sufficient logical rules are there to determine the use of such expression. Under the influence of Alfred Tarski, he included such notions as truth and meaning in his analysis. Kurt Goedel is another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous 'Incompleteness Theorem'. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted. Alonso Church is a noted historian of symbolic logic. Logicians and mathematicians alike are interested in questions related to the decidability of logical and mathematical theories. His main thesis is that there is no general technique to determine or discover the truth or proof of any proposition in arithmetic. In this respect, Church stands opposed to Hilbert who argued that classical mathematics is a consistent system. W.V.O Quine and Curry are two other prominent personalities. While Quine is known for his contribution to the development of set theory, Harkell B. Curry's name is associated with a new branch of logic called 'Combinatory Logic'. It had its birth in H.M.Shaffer's discovery of 'stroke' symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables. Curry proceeded further with Schonfinkel's works with set of operations different from stroke symbol. He introduced what is called the theory of λ – conversion (λ is read 'lamda'), where λ is known as binary operation. Church used this operation to analyze formal systems to which variables belong and to which arbitrary objects can be substituted. Here objects mean the functions in which they stand for arguments. It means that a variable in a system is substituted by an argument. λ – conversion is a theory proposed by Church in connection with such substitutions. In short, symbolic logic is a system of algebraic combination and mechanical substitution of symbols for the purpose of inference. It is the study of symbolic abstractions that captures the formal features of logical inference. C.I. Lewis observes the following characteristics for symbolic logic: the use of ideograms (i.e., signs that stand directly for concepts) instead of phonograms (signs that depict sounds first and indirectly concepts); deductive method and use of variable having definite range of significance. It has mainly two parts: truth-functional or propositional or sentential logic and predicate logic. The former is a formal system in which propositions can be formed by combining simple propositions using sentential connectives, and a system of formal proof in determining the validity of arguments. Predicate logic provides an account of quantifiers in the symbolization of arguments and laws for the determination of their validity.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Examine Boole's contribution to modern logic.

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2. Examine the role played by PM in the 20th century logic.

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3. Contrast Hilbert's and Goedel's views on proofs in mathematics.

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4. What is the significance of Shaffer's and Schonfinkel's studies? Explain.

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2.7 LET US SUM UP

Logic has its roots in Greek civilization. Aristotle systematized the technique of thinking. During medieval ages, lot of research work was undertaken within the limits of Aristotelian system. Modern logic took its birth with Leibniz' work 'Dissertatio de Arte Combinatoria'. Boole's works provided impetus to the growth of symbolic logic. Contemporary symbolic logic begins with de Morgan. Initially, Frege and Russell and later, Russell and Whitehead heralded a new era in symbolic logic. Combinatory logic has its beginning in H.M. Shaffer's work which was later developed by Haskell B. Curry. Today logic and mathematics have become two faces of the same coin.

The history of logic deals with the study of the development of the science of valid inference (logic). Formal logics developed in ancient times in India, China, and Greece. Greek methods, particularly Aristotelian logic (or term logic) as found in the Organon, found wide application and acceptance in Western science and mathematics for millennia. The Stoics, especially Chrysippus, began the development of predicate logic.

Christian and Islamic philosophers such as Boethius (died 524), Ibn Sina (Avicenna, died 1037) and William of Ockham (died 1347) further developed Aristotle's logic in the Middle Ages, reaching a high point in the mid-fourteenth century, with Jean Buridan. The period between the fourteenth century and the beginning of the nineteenth century saw largely decline and neglect, and at least one historian of logic regards this time as barren. Empirical methods ruled the day, as evidenced by Sir Francis Bacon's Novum Organon of 1620.

Logic revived in the mid-nineteenth century, at the beginning of a revolutionary period when the subject developed into a rigorous and formal discipline which took as its exemplar the exact method of proof used in mathematics, a hearkening back to the Greek tradition. The development of the modern "symbolic" or "mathematical" logic during this period by the likes of Boole, Frege, Russell, and Peano is the most significant in the two-thousand-year history of logic, and is arguably one of the most important and remarkable events in human intellectual history.

Progress in mathematical logic in the first few decades of the twentieth century, particularly arising from the work of Gödel and Tarski, had a significant impact on analytic philosophy and philosophical logic, particularly from the 1950s onwards, in subjects such as modal logic, temporal logic, deontic logic, and relevance logic.

2.8 KEY WORDS

Theorem: In mathematics, a theorem is a statement proved on the basis of previously accepted or established statements such as axioms.

2.9 QUESTIONS FOR REVIEW

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- Wilder L., Raymond and Wiley. Introduction to the Foundations of Mathematics. New York: John and Sons Inc, 1952.

2.10 SUGGESTED READINGS AND REFERENCES

1. Discuss the Earliest Contributions to Logic
2. What are the Limitations of Aristotelian Logic?
3. Discuss the History and Utility of Symbolic Logic.

4. Discuss the Rise of Symbolic Logic.
5. Discuss Age of Principia Mathematica (PM).

2.11 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1 Boole's contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work 'An Investigation of the Laws of Thought!' It is in this work that the germs of the 20th century symbolic logic can be traced.

2 The publication of Principia Mathematica by Russell and Whitehead heralded a new era in the history of mathematics and logic. In this work they established that logic is the foundation of mathematics. The term implication acquired a new meaning when new rules of inference were evolved. These rules of inference forced logicians to distinguish implication from entailment. Also this work influenced Emil Post to present the methods of truth-table which is the backbone of mathematical logic. The earlier Wittgenstein was also partly influenced by this work.

3. David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore a consistent system, in Hilbert's analysis is an axiomatized system. Kurt Godel is

another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous ‘Incompleteness Theorem’. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted.

4 Combinatory logic had its birth in H.M.Shaffer’s discovery of ‘stroke’ symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables.

UNIT 3: NATURE OF MODEL LOGIC

STRUCTURE

3.0 OBJECTIVES

This unit titled Nature and Scope of Logic aims at:

• introducing and familiarizing the definition, nature and scope of the subject exposing the students to various definitions of logic.

• discussing the question whether it is an art or a science, a positive science or a normative science

• discussing the extension and scope of logic

3.1 INTRODUCTION

"Reasons are the coin we pay for the belief we hold," so says Schipper in his monumental work on Model logic. But reasons given are not always good enough. With reasoning we produce arguments – some good, some bad – that often get converted in writing. Every argument confronted raises this question: Does the conclusion reached follow from the premises used or assumed? There are objective criteria with which that question can be answered, in the study of logic we seek to discover and apply those criteria. Usually logic is associated with Greek tradition and philosophy. Most of us think logic as a branch of knowledge originated in ancient Greece. But this is not true since as a matter of fact almost all great civilizations developed logic as an academic discipline. Ancient Indians, Arabs, and Chinese made significant contributions to the growth and development of logic. However, our study is restricted logic developed by Europeans over several centuries.

3.2 VARIOUS DEFINITIONS OF LOGIC

The word 'logic' comes from the Greek word logos, literally meaning, word, thought, speech, reason, energy and fire. But in due course of time these literal meanings were given up to make way for more accurate meaning hinting at what we actually learn when we do logic. This is how it came to be understood as a discipline dealing with thought, reasoning and argument at different points of time. It is our experience that emotional appeal is sometimes effective. But it has no place in logic. Only appeal to reason pays effectively in the long run and which can be objectively verified and appraised. One needs to discern the criteria involved in rational method. The goal of the study of logic is to discover and make available those criteria that can be used to test the correctness of arguments. Against this background we shall evaluate various definitions of logic held at different times and their merits and demerits. One of the definitions of logic states that it is the study of reflective thinking. This particular definition was proposed by Susan Stebbing in her work 'A Modern Introduction to Logic'. She, surely, made progress over H.W.B. Joseph who regarded thought in its unqualified sense as the main theme of logic when he wrote 'Introduction to Logic'. However, the fact is that one has to concede in both the cases that the content of logic is essentially psychological and what is psychological is invariably subjective. This position is unacceptable to any student of logic. A clarification is needed on this issue. One of the important topics of logic is what is known as 'Laws of Thought.' There are three laws of thought, law of identity, law of excluded middle and law of of contradiction. On this ground, it is possible to conclude that at least indirectly logic deals with thought. However, this is a mistaken notion. Laws of thought, in reality, have nothing to do with thought. They merely show or demonstrate the nature of statements. Therefore even in this sense thought cannot enter the domain of logic. Another discarded definition of logic states that it is the study of the methods or principles which we use to distinguish good (correct) reasoning from bad (incorrect) reasoning. As it has been claimed 'All reasoning is thinking but all thinking is not reasoning'. There are many psychological processes that are different from reasoning, such as imagining, regretting, day dreaming and so on. There seems to be same laws governing all these activities, but they are not studied by logicians. Reasoning is a special kind of thinking in which problems are solved and conclusions are drawn from premises. The logician is primarily concerned with the correctness of the completed process of reasoning and only with this species of thinking. This definition does not imply that only a student of logic can reason well. Nor does it imply that a student of logic necessarily does it. Just as an athlete need not be aware of the complex processes going on inside his body while he performs the athletic fete, people need not be conscious of the complex logical processes involved in reasoning when they scrupulously perform the task of reasoning. However, a person, who has studied logic, is more likely (there is no rule that he should do) to reason correctly than one who has never thought about the principles involved in logical activity. There are multiple reasons for it. To begin with, a student of logic will approach the discipline as an art as well as a science, and he or she will engage herself in doing exercises in all parts of the theory being learned. It is a continuous practice that will help the student fare better and make him perfect. Second, a significant part of the study of logic consists in the examination and analysis of fallacies, which may be viewed as quite natural mistakes in reasoning. Knowledge of such pitfalls gives an increased insight into the principles of reasoning in general and thereby we can avoid stumbling upon them. Finally, a study of this discipline empowers the student with techniques and methods for testing the correctness of many different kinds of reasoning, and when errors are detected, they are removed at once. Again, problem with this definition is that whatever may be its merit, it is also subjective because reasoning depends upon the person who reasons. If there is no one who reasons, then there is no reasoning at all. Therefore this definition also does not take us far. As an alternative, logic was defined as the science of inference by some logicians. Though this definition is better than the older definitions, even this definition is not free from defect completely. Inference is a special form of mental activity. Its subjective nature becomes obvious when we notice that if there is some one who infers, then there is inference; not otherwise. However, very shortly we notice that inference is not banished altogether from the domain of logic and that it has a definite role to play in the development of logic. If so what is an acceptable definition of logic? Logic concerns with distinction between good argument and bad argument. This itself constitutes the definition or essence of logic. An argument always points to a certain relation between two sets of statements or propositions. One set is called premise or premises and another is called conclusion. If the conclusion follows from the premises, then the argument is said to be good; otherwise bad. How do we know whether the conclusion follows from the premises or not? As in the case of games here also total adherence to rules makes an argument good. Even if one rule is violated the argument turns out to be bad. It only means that conclusion follows from the premises only when all rules are scrupulously followed. At this stage, we introduce a technical word. We say that the premises imply the conclusion if the same follows from the given premises. Therefore implication is the desired relation between the premises and the conclusion. Implication is not something which is brought from outside. It is latent in the premises only. It is left to the intellect of human being to discover or to extract what is latent. Implication is objective and, therefore, man-independent because if it exists, it exists independent of any thinking mind. No amount of effort on the part of thinking minds can impose implication when it does not exist. It can only be discovered, but cannot be created. The process of discovering what is latent is known as inference. Logic is not concerned with the process as such, but with the end product of process, i.e., presence or absence of implication. This will bring us to the crucial distinction to be made. Inference can be valid or invalid. If inference has its basis in implication, then it is valid. On the other hand, if it does not enjoy the support of implication, then it is invalid. However, there is nothing like valid or invalid implication. Either there is implication or there is no implication. That is all.

Secondly, statements imply; they do not infer. On the contrary, humans infer; they do not imply. Therefore any error lies only in human activity. No error can be discerned in the relation between statements. In the third place, implication without inference (valid) is possible, but valid inference without implication is neither possible nor plausible. This sharp distinction has its tell-tale impact. Contrary to inference which is man-dependent implication is man-independent. Suppose that logic is defined as a study of inference. Then it becomes subjective. If I infer then only there is logic; otherwise not. On the contrary, if implication replaces inference, then logic becomes man-independent and hence objective. Rivalry between subjective and objective elements now surfaces. If knowledge is to be viewed as objective, then logic, automatically, ought to remain objective. Therefore implication replaces inference when we are concerned with the subject matter of logic. Though inference loses its place in this scheme, philosophers like Russell continued to use 'inference' only. Later we will learn that we have only rules of 'inference' but not rules of implication. The point to be noted is that in all these cases inference, paradoxically, means implication only. It is very important that this point is borne in our mind throughout our study of logic.

3.3 TWO TYPES OF LOGIC: FORMAL AND MATERIAL

Traditionally logic has been classified into two types 1) Formal and 2) Material logic. Formal logic is otherwise known as deductive logic and material logic as inductive logic. Formal logic is concerned with the form or structure of argument whereas material logic is concerned with the matter or content of argument. When matter is irrelevant, material truth also is irrelevant. What matters in deductive logic is formal truth. By formal truth we mean logical relation between the premises and the conclusion. It is possible to know this kind of truth without knowing the content of the argument. In this case, it is sufficient if the argument follows the rules of the game. This whole explanation can be put in a nut-shell in this manner. An argument consisting of only true propositions can very well be invalid whereas an argument consisting of only false propositions can very well be valid. It also means that in our study of deductive logic it is possible to know whether an argument is valid or not without knowing the contents of the argument (and many times this is what precisely happens) provided we are in a position to decide whether the argument has followed all the rules are not. However, the case of material logic is different. In this case it is possible to judge the truth or falsity of the conclusion only when we know what the argument is all about. What is more important than the previous statement is the controversy surrounding the relevance of rules. The burning question is whether there is anything like rule or rules governing the structure of inductive argument (for more details see, 1.4 of block 2). Suppose that there are no rules regulating inductive arguments as maintained by some philosophers. Then inductive arguments are neither valid nor invalid. If so, what is its status? A question like this is easier asked than answered. Attempts to answer this question occupy a good deal of discussions on inductive logic.

3.4 LOGIC: SCIENCE OR ART?

Questions have been raised on the issue whether logic is a science or an art or both. Let us stay for a while on this problem. In ancient times science just meant a systematic study of anything. But today the term science has developed into a discipline distinct from several other activities of mankind. Science has been defined as that branch of knowledge which aims at explanation of phenomena. Used in this technical sense, logic is no science at all. Does this mean that logic is an art? Art is concerned with doing something. Logic, if defined as an art, is so only in derivative sense. In order to decide whether or not logic is an art we have to consider the aim of logic. Is the aim of logic to give us knowledge about valid argument forms or to make us better thinkers? No one will deny that a study of logic results in improving our reasoning ability. But there is a restriction. Just like a moralist who may not himself be moral as a person, a logician may not be logical in his reasoning. We can say that the effect of such a study is the acquisition of knowledge regarding valid argument forms. It is not for logic to consider whether or not this knowledge is put into practice. In view of this feature we can say that logic is a science and not an art. It is a science not in the technical sense, but in a general sense.

3.5 LOGIC: POSITIVE SCIENCE OR NORMATIVE SCIENCE?

Granted that logic is a science, what type of science is it? Science has been classified into two types, viz., 1) positive Science and 2) normative Science. Positive science describes what the case is. Normative science, on the other hand, tells us what ought to be the case. Let us now examine whether logic is a positive science or a normative science. Some logicians consider logic to be a formal science and regard it as a normative science. Just like object thought is made up of form and matter. According to Latta & Macbeath 'the form of thought is the way in which we think of things, the matter of thought is the various particular objects we think of. A form is something which may remain uniform and unaltered, while the matter thrown into that form may change and vary. A normative science attempts to find out the nature of forms (standards) on which our judgments of value depend. Normative sciences have before them a standard with reference to which everything within the scope of science is to be judged. A normative science gives us judgments of value, i.e., it tells us what ought to be the case. Logic has an important normative aspect; but a norm or ideal in logic has a special meaning. The main business of logic is to discover the general conditions on which the validity of inference depends. In our discussion of logic we try to force these conditions on the way of arguing. We do so because there are certain objective relations between statements. This means that statements must possess a certain structure and there must be certain objective relations between them if our inferences are to be valid. These structures of statements and their mutual relations are pure forms, which serve as norms in logic. Traditional logicians while considering logic to be a normative science meant that it is a science concerned with those principles which ought to be followed in order to attain the ideal of truth. Some other logicians consider logic to be a descriptive science or a positive science and not a normative science since it does not lay down any norm for thinking. Its nature is description as it aims at describing and classifying various types of arguments. In fact the classification of sciences into positive and normative cannot be applied to logic. Logic cannot be characterized either as positive or as normative science. If logic were a positive science, it would merely describe different argument forms. Logic however, does not do so. The logician aims to build a deductive system whose elements are logically true propositions (tautologies). These propositions are purely formal and hence have no reference to context. Similarly, logic cannot be considered normative. It does not search for principles on which value judgments depend. In fact, the starting point for logic is our ability to distinguish between valid and invalid arguments. The logician only makes explicit the principles involved in valid arguments. This discussion reveals that positive-normative distinction is not relevant in the context of logic.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) Bring out merits and demerits of various definitions of logic.

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2) Is logic a Positive science or a Normative Science? Substantiate your position.

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3.6 LOGIC AND OTHER DISCIPLINES

Logic as a discipline has wide scope and this will be clear if we examine its relationship to various empirical and social sciences. Logic is closely associated with almost all disciplines. Some are very significant. Therefore a cursory reference to some of them is desirable.

Logic and Epistemology:

Epistemology is that branch of philosophy which deals with theories of knowledge. It investigates the structure, conditions, sources as well as limitations of human knowledge. Epistemology, though, is not a formal science like logic since it must deal subjective entities like belief it does make use of logic and its methods widely to form theories about it. In fact there is a subdivision within epistemology called epistemic logic which specifies the limits of logical norms applicable in epistemic situations. Though logic and epistemology are interrelated, we cannot attribute any genus – species relation between the two. Logic is the science of reflective thinking in so far as implications are concerned. The province of logic is confined to certain formal methodologies. Epistemology consists of a number of cognitive affairs which goes beyond logic. Similarly logic too extends outside the concerns of epistemology.

Logic and Metaphysics:

Traditionally, the subject matter of metaphysics is regarded as the nature of Being or Reality. Since Greek times metaphysics has been conceived as the mother of all knowledge and it is this subdivision of philosophy which examines every presupposition of various sciences. For instance, physics assumes the existence of matter, motion, force, time and space. It is metaphysics which takes upon itself this task of examining these presuppositions of various sciences. The basic assumption of logic is that thought gives knowledge. It is necessary that we enquire into this very presupposition. In this endeavour metaphysics comes to our aid. Again it is common to make a distinction between real and unreal. But inquiry into the basic nature of this distinction is not common. Metaphysics deals with this problem as well. Not only does it analyse the basis of all sciences, but also provides a criterion of reality. Logic in fact stands between metaphysics and science. Abstraction of the bases of the principles of science is done through logic which bridges the gap between metaphysics and sciences.

Logic and Psychology:

Tradition stipulates that logic and psychology are related on the basis of the assumption that thought is a common factor to them. However, a crucial point is missing in this correlation. The traditional approach is something like this. Psychology takes up the study of origin and evolution of thought process by examining the functions of animals, infants, abnormal persons, criminals etc. Its main concern is mind and thought is a mental process. Logic gets confined to the study of inferential thinking of only normal adult human beings. Again while logic attempts to abstract the forms in which human mind thinks, psychology studies the actual process of thinking. The forms of thinking which logic abstracts from our thought processes are not events in our mind and, therefore, are not of interest to psychologists. Being a formal science, logic looks upon those principles as regulative elements of reflective thinking. Psychology is concrete because its subject – matter is concrete, i.e., actual psychological events. Logic is abstract because its subject matter is abstract, i.e., forms of reflective thinking. Therefore in one sense they are related and in some other sense they are poles apart.

Ironically, this is just a matter of history of psychology as well as logic because today psychology does not regard mind as the topic of its concern and thought is no longer reckoned as mental. It is at once transformed into a sort of neurological process though its subjective nature remains unaltered. Only in this sense psychology studies thought. And logic is anything but a study of thought. Hence it is really obsolete to relate logic and psychology. Therefore logic and psychology are distinct disciplines and have nothing in common. However, we can remark that there is something logical in psychology though there is nothing psychological in logical enterprise. This is so because no science can afford to be illogical and, admittedly, at least some sciences can progress without recourse to psychological elements.

A question is frequently asked; which one has wider application; logic or psychology? This is an unanswerable question. In one sense the province of psychology is wider than that of logic since the former studies the entire activities of the human mind. In a different sense logic is wider than psychology because the latter follows logical principles while dealing with its own subject matter. The two sciences are mutually complementary.

Logic and Language:

Language is only a means of expression, yet the nature of language affects logical thinking. Just as the success of an operation depends upon the quality of surgical instruments apart from the skill of the surgeon, the quality of the argument depends upon not merely the validity of the forms of thinking the agent resorts to, but on the language in which the arguments are expressed as well. Natural language performs multiple functions, like conveying information, evoking emotions, stimulating action, making reference and so on. The structure of natural language is so constituted that it enables the language to perform these diverse functions. However, language of logic needs to convey only information. Hence it calls forth the use of emotively neutral language. Logicians take extra care in using plain and non-sophisticated language so that they just convey information, which is either true or false. Logical statements pronounce that something is or is not the case. For instance, 'Atom has been split' is a factual statement which carries a definitive truth-value. Logic demands statements which convey exact information through a neutral use of language. Language is so subtle and complicated an instrument that we often lose sight of the multiplicity of its uses. But there is real danger in our tendency to over simplify. On the staggering variety of uses of language some order can be imposed by diving them into very general categories: the informative, the expressive and the directive. Among these three uses, logic is concerned only with the informative use of language. Many philosophers, however, have claimed that the structure of logic and language are identical. Therefore, a better understanding of logic depends upon the elimination of ambiguity and vagueness of language.

Logic and Physical Sciences:

Of late, science and scientific culture seem to shape human life. The goal of science is to study the natural events of various types and discover generalization regarding them. The generalisations are utilized to yield comprehensive theories about the working of nature. The procedure of science involves both observation of facts and reflective thinking. The principles of logic help science to analyse the observed facts and draw valid conclusions from them.

Logic and Mathematics:

Let us briefly dwell on the background before proceeding further. Though the beginnings of modern logic are found in the writings of Leibniz, it was not until the end of the nineteenth century that logic discovered new path of development. The shift in track was partly due to certain topics in mathematics which received the impetus and partly due to the discovery of paradoxes. These developments resulted not just in the overlapping of logic and mathematics, but at some point of time, it became 'extremely difficult to draw a non-arbitrary line between logic and mathematics'. In this section, only cursory reference can be made to important milestones which led to constant interplay between logic and mathematics.

The ball was set rolling by George Boole when his work on 'The Mathematical Analysis of Logic' was published in 1847. The essence of his work was with his treatment of the logic of classes. This was followed by George Cantor's investigations on theory of sets. What made Cantor's work on theory of sets significant were his studies in analysis in general, and theory of trigonometric series in particular. However, the required breakthrough was provided by Gottlob Frege when he attempted to base mathematics on pure logic. In his own words, arithmetic is only a development of logic. Not only arithmetic became an extension of logic, but also due to the discoveries of non-Euclidean schools of geometry and certain paradoxes by Russell, Cantor and others at a later stage, mathematics itself was regarded as an extension of logic and this thesis came to be known as Frege-Russell thesis.

This extension was described by Russell and Whitehead in their preface to the 'Principia Mathematica' as backward extension, thereby meaning extension to roots. G. Peano tried a different route to connect mathematics and logic. Instead of trying to secure a sound base in logic to mathematics, he analysed the methods of mathematics which were structurally similar to the calculus of logic and in this way he tried to link the two. None of these attempts aimed at ‘mathematicising’ logic so much as ‘logicising’ mathematics. Consequently, logic became the foundation of mathematics. Serious reservations against this theory came only from two quarters. Kronecker questioned the ideas of Cantor only to challenge the ‘ostensible’ essence of mathematics because he believed that Cantor’s theory was not mathematics but sort of mysticism, a view partly endorsed by Cantor himself. Poincare was another philosopher who reacted in the same spirit to Zermelo’s axiomatic set theory. Poincare’ argued that the nature of natural number system is such that it is incapable of being reduced to logic. He was more emphatically opposed to ‘reducing’ mathematical induction to logic. Surprisingly, he argued that mathematical concepts should be built up inductively by proceedings from ‘particular’ to ‘general’. Perhaps he subscribed to the view that induction is not logic. A brief reference to of mathematical induction mentioned above is relevant. Mathematical induction is a misnomer because, in reality, there is no inductive element at all involved here, even though the principle proclaims that ‘every natural number has a successor’, i.e., if n is a natural number, then n+1 is also a natural number. This is the essence of mathematical induction. This theorem involves rigorous logical proof which is essentially deductive in nature with no semblance of inductive inference. It should be mentioned that Poincare’ did not oppose mathematics following deductive model.

Following a certain logical method is not the same as reducing a certain science to logic. Poincare’ was only against making the latter. If we go by the modern definition of mathematics as the science of formal proof or logical demonstration, then the relation between logic and mathematics becomes very intimate. Both logic and mathematics are formal sciences. They deal with relations between propositions which are independent of the content of the propositions. In arithmetic, for instance, we may use numbers to count anything. What we actually count makes no difference to counting. Thus two plus two will be four whether we add books, balls, tables or anything else. Since the relations with which logic and mathematics deal are independent of content these sciences are able to use symbols in place of words.

Also, both logic and mathematics deal with relations which are applicable to actual as well as possible objects. Further, both logic and mathematics are deductive in character. They begin with certain axioms and deduce conclusions from them. Moreover, the method of both is a priori. Though both logical and mathematical operations may take place with reference to any empirical entity, knowledge of the principles of these disciplines is not gained by observation or sense experience. Such knowledge is called ‘a priori’, i.e., independent of experience.

3.7 DEDUCTIVE AND INDUCTIVE LOGIC

Traditionally arguments have been classified into two types, viz., deductive and inductive arguments. Accordingly there are two divisions of logic, viz., deductive logic and inductive logic. Deductive logic has arguments that consist of premise or premises and a conclusion. In a deductive argument the conclusion necessarily follows from the premises. Furthermore, it is the characteristic of the deductive argument that if one accepts the premises one has to accept the conclusion. Such arguments are available in mathematics and geometry. Deductive argument is not concerned with truth and falsity, but it is concerned with validity and invalidity or consistency and inconsistency of arguments. Validity and invalidity are characteristics of arguments whereas truth and falsity are characteristics of propositions. There is another kind of argument which is known as inductive argument, the concern of inductive logic. According to one group of philosophers, inductive arguments are found in empirical sciences such as physics, sociology, psychology etc. This view is hotly debated. Law of causality constitutes the very basis of inductive arguments. Generalisations and predictions are the objectives of inductive arguments. Generalization is an important parameter of inductive logic. Therefore a brief description of what it means is necessary. Suppose that I observe ten crows which are black. Then I jump to the conclusion that all crows are black without observing other crows. Therefore the conclusion includes and goes beyond observation. Such conclusion is called generalization. Therefore mere acceptance of the truth of premises do not warrant acceptance of the truth of conclusion. The conclusion is rendered probable because it may be true or false. This is how probability enters the field of inductive logic.

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) State the relation between logic and language.

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2) Distinguish between deductive and inductive arguments.

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3.8 LET US SUM UP

Humans are endowed by nature with powers of reasoning. Logic is the study of the use of those powers. In the study of logic we come to recognize our own native capacities, and practice helps us to sharpen them. The study of logic helps one to reason well by illuminating the principles of correct reasoning. Correct reasoning is useful wherever knowledge is sought. Whether in science, politics or in the conduct of our private lives, we use logic in reaching defensible conclusions. In formal study we aim to learn how to acquire reliable information and how to evaluate competing claims for truth. Various definitions of logic were discussed and also types. Questions regarding the status of logic as an academic discipline were addressed subsequently. Arguments for and against logic as a science/ art, and logic as a positive science/ normative science, were discussed. The relevance scope of logic was examined by looking into the relation logic has with various other branches of knowledge. At the close of the unit, deduction and induction, the two major types of logic have been introduced to the student.

3.9 KEY WORDS

Logos: Logos is an important term in philosophy, analytical psychology, rhetoric and religion. Heraclitus (ca. 535–475 BCE) established the term in Western philosophy as meaning both the source and fundamental order of the cosmos. The sophists used the term to mean discourse, and Aristotle applied the term to rational discourse. After Judaism came under Hellenistic influence, Philo adopted the term into Jewish philosophy. The Gospel of John identifies Jesus as the incarnation of the Logos, through which all things are made. The gospel further identifies the Logos as divine (theos).

Positive Science: In the humanities and social sciences, the term positive is used in a number of ways. One usage refers to analysis or theories which only attempt to describe how things are, as opposed to how they should be. In this sense, the opposite of positive is normative. An example for positive, as opposed to normative, could be economic analysis. Positive statements are also often referred to as descriptive statements.

3.10 QUESTIONS FOR REVIEW

1. Discuss the Various Definitions of Logic.
2. What are the Two Types of Logic? Discuss about Formal and Material
3. Discuss Logic: Science or Art?
4. Discuss Logic: Positive Science or Normative Science?
5. Relate Logic and Other Disciplines
6. What is Deductive and Inductive Logic?

3.11 SUGGESTED READINGS AND REFERENCES

- Copi, Irving M. & Cohen, Carl. Introduction to Logic. New Delhi: Prentice Hall of India, 1997.
- Copi, Irving. M. Symbolic Logic. Delhi: Prentice Hall of India, 2005.
- Das, G. Logic: Deductive & Inductive. Delhi: King Books, 1684

3.12 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) Bring out the various definitions attributed to logic? Logos – literal meaning: word, thought – eventually logic acquired a technical meaning – defn: Study of methods and principles which we use to distinguish good (correct reasoning) from bad (incorrect) reasoning – also defined as science of the laws of thought – again as science of reasoning.
2) Logic: is it a positive science or normative science? Substantiate your position. Positive science: describes what is the case – Normative science: tells us what ought to be the case – Formal science is that which takes up the form of the subject content for study – Normative science follows the norms – gives judgments of value – some logicians characterize it as positive science as well for its nature is description. It aims at describing and classifying various types of reasoning.

Check Your Progress 2

1) What is the relation between logic and language? Language affects logic – Natural language is an inconvenient tool to operate logical functions – Natural language being endowed with potency to attend divergent functions cannot get confined to the single function of conveying information – hence it calls forth the use of emotively neutral language – three functions of language: informative, expressive and directive – of these only informative use is conducive to logic.

2) Distinguish between deductive and inductive logic Historically logic has been divided into two – deductive and inductive. In deductive logic an argument's conclusion necessarily follows from the premises. Such arguments are available in mathematics and geometry. In a deductive argument we are concerned with validity and invalidity. Inductive logic has arguments that are found in empirical and social sciences. Generalizations and predictions are the objectives of inductive arguments.

UNIT 4: LOGICAL INTERCONNECTIONS BETWEEN NECESSARY, THE IMPOSSIBLE AND PERMITTED

STRUCTURE

4.0 OBJECTIVES

After this unit, we can able to know:

- To know about the Kripke on a posteriori Necessities and The Deduction Model
- To discuss about the Epistemic Issues Pertaining to Kripke's Work
- To describe Rationalist Theories

- To describe Counterfactual Theories

4.1 INTRODUCTION

Whereas facts about what is actual are facts about how things are, facts about modality (i.e., what is possible, necessary, or impossible) are facts about how things could, must, or could not have been. For example, while there are in fact eleven players on a soccer team, there could have been thirteen, though there couldn't have been zero. The first of these is a fact about what is actual; the second is a fact about what was possible, and the third is a fact about what is impossible. Humans are often disposed to consider, make, and evaluate judgments about what is possible and necessary, such as when we are motivated to make things better and imagine how things might be. We judge that things could have been different than they actually are, while other things could not have been. These modal judgments and modal claims therefore play a central role in human decision-making and in philosophical argumentation. This entry is about the justification we have for modal judgments.

Most of the time, we encounter what might be called ordinary modal judgments, such as the following:

i. Although I am a philosopher, I could have been a musician.

ii. Not only does 2 + 2 = 4, it is necessary that 2 + 2 = 4.

iii. Not only is it the case that nothing is red and green all over at the same time, it is impossible for something to be red and green all over at the same time.

iv. Although the table is not broken, it could have been broken.

v. Even though the cup is on the left side of the table, it could have been on the right side.

However, philosophers often, in the course of an argument, formulate what might be called extraordinary modal judgements; these typically are about some special philosophical concept relevant to the discussion. Here are some examples:

St. Anselm

Necessarily: God exists.

Descartes

It is possible for the mind to exist without the body.

Berkeley

It is impossible for anything to exist unperceived.

Now a modal argument is one in which either a premise or the conclusion is an ordinary or an extraordinary modal judgment. Thus, in modal arguments, we reason about what is necessary, possible, or impossible, or about what might, must, or could not be the case. Modal arguments can therefore be found both inside and outside of philosophy (within philosophy many important philosophical positions are in fact modal positions). Assuming that a modal argument is valid (i.e., the premises validly imply the conclusion), then the evaluation of a modal argument focuses on whether the premises are justified. The question then arises: how does one show that a modal premise of a modal argument is justified?

Philosophers have long been interested in how a modal claim can be known, justified, or understood. The philosophy of modality is the area in which one studies the metaphysics, semantics, epistemology, and logic of modal claims—that is, claims about what is necessary, possible, contingent, essential, and accidental. Epistemology is the general area of philosophy in which one studies the nature of knowledge. The central questions of epistemology concern: (i) what it is to know something, (ii) what it is to be justified in believing something, (iii) what it is to understand something, and (iv) what are the means by which we can come to possess understanding, justification, or knowledge. Within the philosophy of modality one finds the sub-discipline known as the epistemology of modality. The central question of this field is:

How can we come to know (be justified in believing or understand) what is necessary, possible, contingent, essential, and accidental for the variety of entities and kinds of entities there are?

This is similar to the central questions found in the epistemology of mathematics and morality, where one inquires into, the nature of mathematical knowledge or moral knowledge. Special interest in modal epistemology (another name for the epistemology of modality) often derives from the following contrast between knowledge of the actual and knowledge of what could have been and could not have been the case.

In general, perception of the actual world can guide us to knowledge of realized possibilities, possibilities that are actual. For most philosophers hold that given that what is actual is possible, knowledge of actuality can inform us of knowledge of some possibilities. However, actuality appears to be an insufficient guide to what is: (a) merely possible, since the possibility is not realized, or (b) impossible, since what is actually the case does not tell us what could not be the case. To better understand this phenomenon, consider a cup, cc, located at LL at time tt. The following line of reasoning illustrates the central question and its special interest in the case of ordinary possibilities.

- Actual world fact: cc is at LL at tt, and SS perceives that cc is at LL at tt.
- Knowledge of actuality: SS knows that cc is at LL, since SS perceives cc at LL and there is no reason for SS to believe that their perception of cc at LL is misguided.
- Actuality-to-Possibility Principle: If PP is actually true, then PP is possibly true, since realized possibilities are evidence of possibility.
- Knowledge of Realized Possibilities: SS can know that it is possible for cc to be at LL through derivation from the actuality-to-possibility principle and perception of the actual world fact.
- Non-Actual/Unrealized Possibility Datum: cc could have been at L∗L∗, a location distinct from LL, at tt.
- SS believes that cc could have been at L∗L∗ at tt, and SS can come to know that cc could have been at L∗L∗ at tt.
- Epistemic Question: How does SS know that cc could have been at L∗L∗ at tt?

With respect to the epistemic question, all of the following have been proposed as potential answers:

- Perception: even though cc is not at L∗L∗. SS sees that cc could be at L∗L∗.
- Intuition: even though cc is not at L∗L∗, SS has a non-sensory based intuition that cc could be at L∗L∗ when SS entertains the question: could cc have been at L∗L∗?

- Conceivability: SS can conceive of a scenario in which cc is at L∗L∗. SS derives justification for believing that cc can be at L∗L∗ from conceiving of it.
- Imaginability: Were SS to imagine a process whereby cc moved from LL to L∗L∗, SS would not arrive at a contradiction. So, SS is justified in believing that cc could have been at L∗L∗ on the basis of imagining the movement.
- Deduction: SS can deduce from knowledge of what cc is fundamentally and the relevant details about location L∗L∗ that cc could have been at L∗L∗, since what cc is fundamentally is not incompatible with it being at L∗L∗.
- Theory: From SS's knowledge of what cc is, as well as the relevant facts about the location of L∗L∗, SS can come to know that cc could have been at L∗L∗.
- Similarity: From SS's prior observation of objects relevantly similar to cc, as well as their actual locations and movement, SS can come to know that cc could have been at L∗L∗.

In addition to these theories, one overarching idea is that they can either be offered as part of a uniformity account or as part of a non-uniformity account of modal knowledge. The uniformity view holds that there is only one single route to modal knowledge at the most fundamental level of explanation. The non-uniformity view maintains either that different people can come to know the same modal truth through different routes or that at the fundamental level of investigation there must be more than one route to modal knowledge.

In addition to the central question there are three other main questions of interest.

Modal Sorting:

how can we knowledgeably sort necessary truths from essential truths and contingent truths?

At least one point of interest in the sorting question derives from work in the metaphysics of modality. Necessity and possibility are interdefinable, PP is necessary when it is not possible that not-PP. However, some such as Fine (1994), have argued that essence cannot be defined in terms of necessity. This leads us to the question: how can we sort the essential from the necessary?

Modal Skepticism:

what are the limits of modal knowledge?

At least one point of interest in the skeptical question derives from work on the range of modal knowledge. All theories of modal knowledge should be able to account for ordinary cases. However, some, such as Van Inwagen (1998), have presented skeptical arguments about extending modal knowledge to a variety of exotic philosophical claims.

Modal Architecture/Epistemic Priority:

given that there is a distinction between necessity, possibility, and essence, is knowledge of one more fundamental than knowledge of the others? For example is our knowledge of necessity more fundamental than our knowledge of possibility and essence, and additionally a pathway to our knowledge of both possibility and essence?

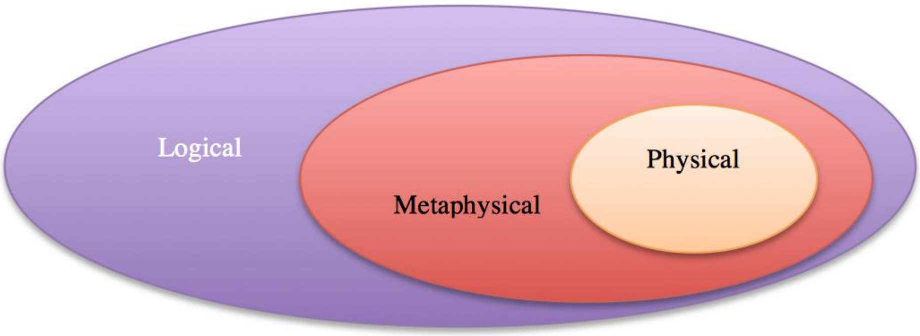
At least one point of interest in the architecture/epistemic priority question derives from work on the proper route to modal knowledge. Bob Hale (2003) has drawn an important distinction between necessity-first and possibility-first approaches to modal knowledge. A necessity-first approach holds that we first arrive at knowledge of necessary truths, and then derive knowledge of possibility through compatibility with knowledge of necessity. A possibility-first approach holds that we first arrive at knowledge of possible truths, and then aim to determine what necessary truths hold.

It is important to take note of two points about general inquiry in the epistemology of modality. First, the field is typically concerned with investigating (i) alethic modality (modality concerned with what could have been true), as opposed to epistemic modality (modality concerned with what might be true in an epistemic sense of "might") or deontic modality (modality concerned with what might be done in some normative or evaluative sense). Second, (ii) the investigation centers on metaphysical modality, as opposed to logical or physical modality.

For those that accept the reality of metaphysical inquiry, metaphysical modality is often understood as being the modality concerned with metaphysics as opposed to logical modality, which concerns itself with

Notes

logical relations or physical modality, which concerns itself with physical relations. In addition, on the standard model of the relation between these kinds of modalities the logical possibilities are the most inclusive; they include any proposition that sheer logic leaves open, no matter how otherwise impossible it might be. The metaphysical possibilities are the logical possibilities that are also allowed by the natures of all of the things that could have existed. The physical possibilities are the logical and metaphysical possibilities that are also allowed by the physical laws of nature. On the standard model, the following nesting relation holds:



This entry will focus on a selection of theories in the epistemology of modality.

4.2 KRIPKE ON A POSTERIORI NECESSITIES AND THE DEDUCTION MODEL

Contemporary analytical debates in the epistemology of modality often take Saul Kripke's (1971, 1980) defense of a posteriori necessities (necessities that are knowable only through sense experience, and not by way of abstract reflection alone) and his deduction model of how we arrive at knowledge of them as a point of departure. In order to better understand what an a posteriori necessity is, it will be important to first introduce the central idea of possible worlds semantics (PWS). Consider the following claims:

i. It is possible that P. For example, although there are 15 people in the room, it is possible that 20 are in the room.

ii. It is necessary that P. For example, not only are whales mammals, it is necessary that whales are mammals.

Now ask: under what circumstances are possibilities and necessities like (i) and (ii) true? According to (PWS), (iii) and (iv) provide the truth-conditions for statements of possibility and necessity.

iii. "It is possible that P" is true just in case P is true in some possible world. Thus, "it is possible that 20 people are in the room" is true just in case in some possible world "20 people are in the room" is true.

iv. "It is necessary that P" is true just in case P is true in all possible worlds. Thus, "it is necessary that whales are mammals" is true just in case in all possible worlds "whales are mammals" is true.

Possible worlds are complete alternative realities; they are ways that the whole of reality might have been. Philosophers have various theories of their nature. (For more about them see the possible worlds entry.) With (PWS) in place an a posteriori necessity is a statement that is true in all possible worlds, and what makes it a posteriori is that it is knowable only by empirical investigation of the actual world. The two most commonly discussed examples are the necessity of Hesperus being identical with Phosphorus, and the necessity of water being identical to H_2O. The former case concerns the celestial body Venus, which is picked out by both "Hesperus" and "Phosphorus". The latter example has to do with theoretical identifications in science, cases in which scientists provide a theoretical identification of a natural kind, such as water, gold, light, or heat by capturing its underlying nature or essence through scientific investigation.

It is uncontroversial that we did, and could only have, come to know that Hesperus = Phosphorus or that water is identical to H_2O through empirical discovery. However, controversially, it is argued by Kripke that these claims involve (a) identity statements between rigid designators (terms that pick out the same thing in all possible worlds in which they have reference), and (b) because they are identity statements between rigid designators, the entities they pick out will be identical in all possible worlds in which the terms have reference. His arguments rely in part on his proof of the necessity of identity. Historically, a posteriori necessities were thought to be theoretically impossible. This is largely due to the work of Kant, in his Critique of Pure Reason, and subsequent empiricists, such as A.J. Ayer, that critiqued Kant’s view. Originally, Kant thought that there could be both analytic (non-informative) and synthetic (informative) a priori truths. Later empiricists argued that the class of synthetic a priori truths (“synthetic” roughly in that they are genuinely informative, not self-evident, and “a priori” roughly in that they are known on the basis of purely rational reflections) was incoherent. (For more about a priori justification see the entry on a priori justification and knowledge). As a consequence of these arguments, in the mid 20th century many philosophers thought that the following equivalences were true:

i. A statement S is a priori if and only if S is necessary.
ii. A statement S is a posteriori if and only if S is contingent.

Kripke’s 1970 lectures, later published as Naming and Necessity (1980), provided a serious challenge to both (i) and (ii). Where “□☐” stands for “it is necessary that”, in his (1971) he offered the following picture of how we can arrive at knowledge of an a posteriori necessity:

First, it is argued that some sort of fact is necessary, if true: (P→□P)(P→☐P).

Second, that the relevant fact is known to be true by empirical investigation: P.

Third, by deduction from (1) and (2) we arrive at a necessary truth, □P☐P, that is known a posteriori because empirical investigation is how the premise P is known.

The first premise in the deduction of an a posteriori necessity involves some necessity-generating principle, a principle that moves from some sort of fact, typically a non-modal fact, to the claim that the fact is necessary. Kripke thought that these principles were usually arrived at through a priori philosophical reflection. Plausible, and often discussed, examples of necessity-generating principles are:

i. The necessity of identity, which maintains that true identity claims are necessary. For example, it is necessary that water = H_2O, since water = H_2O, and both “water” and “H_2O” are rigid designators.
ii. The necessity of origins, which maintains that the originating matter of a given kind of thing is necessary for its existence. For example, given that a table t is wholly carved from a block of wood m, it is

necessary that t originated from m—nothing could be t that did not originate from m. Or, given, that Sheba originated from gamete g, the product of sperm s and egg e, nothing could be Sheba that did not originate from g.

iii. The necessity of fundamental kind, which maintains that the fundamental kind that an entity falls under is necessary for its existence. For example, given that a particular table t is fundamentally a material object, it could not have been non-material. Or, given that a particular organism is a biological kind, such as Sheba being a human being, she could not have been a non-biological kind, and additionally could not have failed to be human.

The second premise in the deduction of an a posteriori necessity is a specific a posteriori truth, a truth that is discovered on the basis of empirical investigation. Given the examples above, the relevant claims would be that, in fact: water = H_2O, t originates from m, Sheba originates from g, t is a material object, Sheba is a biological kind, and Sheba is a human.

From the first and second step a specific a posteriori necessity is deduced. For example: necessarily water = H_2O, necessarily the table originates from its original wood, necessarily Sheba originates from g, necessarily the table is a material object, necessarily Sheba is a biological kind, and necessarily Sheba is a human. In general, learning a conclusion by an argument is a species of a posteriori knowledge just in case at least one premise is known a posteriori. In sum, even though the deduction of an a posteriori necessity involves, as Kripke claims, an a priori known necessity generating principle, because the important fact is known a posteriori, the conclusion is both necessary and a posteriori.

As a generalization of Kripke's model it should be noted that there is no reason why one could not come to know a necessary truth through pure a priori deduction. For example, consider the following:

1. If 2 + 2 = 4, then it is necessary that 2 + 2 = 4 because mathematical truths are necessary truths.
2. 2 + 2 = 4.

therefore

3. It is necessary that 2 + 2 = 4.

In this case, if (1) and (2) can be known a priori, the conclusion drawn on the basis of (1) and (2), will be an a priori necessity.

4.3 EPISTEMIC ISSUES PERTAINING TO KRIPKE'S WORK

In addition to Kripke's seminal work, there are four epistemic issues in the epistemology of modality that are frequently discussed. The first two are reactions to Kripke's work, which challenge the success of his reasoning. The latter two derive from considerations concerning the structure of possible worlds semantics.

4.3.1 The Problem of a posteriori Necessities

It is prima facie plausible to think that all modal knowledge is in principle a priori, since at least perception of actuality cannot provide one with knowledge of mere possibility and necessity. For example, if conceivability is taken to be an a priori exercise, and it is linked to possibility, then it is plausible to think that a priori conceiving that P provides one with a priori justification for believing that P is possible. Likewise, finding P inconceivable provides one with a priori evidence that P is impossible. While this might seem to be the only way that such knowledge can be discovered, this simple thought is challenged by Kripke's arguments for the existence of a posteriori necessities. The problem is discussed in detail in Yablo's (1993): Is Conceivability a Guide to Possibility? One of the main problems facing contemporary a priori accounts of the epistemology of modality concerns the existence of a posteriori necessities. Recall that an a posteriori necessity is a statement, such as the identity statement "Water = H_2O", that is metaphysically necessary, yet knowable only a posteriori. As a consequence, a priori accounts face the following potential situation:

1. To X it seems that P is possible on a priori grounds, such as through conceiving of a scenario S or imagining a situation in which P appears true.
2. Q is necessary and knowable only a posteriori.
3. Q implies that P is necessarily false.

(1)–(3) forces an initial question: if there are a posteriori necessities, how can one have a priori knowledge of modality? Sure one might be able to have it in cases of pure a priori reasoning, such as with respect to mathematical knowledge. But how can one's a priori conceiving of a situation in which, for example, water is present without hydrogen provide one with evidence, sufficient for knowledge, for the claim: it is possible for water to be present without hydrogen? For all one knows they have conceived of a situation or were able to conceive of a situation in which P appears to hold because they do not know the relevant facts which make P inconceivable, since those facts are only knowable a posteriori. Surely one can conceive of a situation in which water does not contain hydrogen, if they simply fail to know that water is H_2O. But why consider that situation to be a situation in which water is present, as opposed to some superficially similar substance?

The initial question is explored in further detail in the literature along side the following questions. Given that knowledge is distinct from justification, and is also a stronger relation than justification, do a posteriori necessities pose a problem for a priori justification about modal truths or only for a priori knowledge? Do a posteriori necessities render a priori reasoning merely fallible or also completely unreliable?

4.3.2 The Relevant-Depth Problem

Van Inwagen (1998), taking note of Yablo's (1993) account of what it is to conceive something, discusses what has come to be a fundamental challenge for theories involving conceivability and imaginability. The problem presented by van Inwagen is related to the problem of a posteriori necessities. Van Inwagen's goal is to present a limited form of skepticism about modal knowledge. He is not a skeptic about all modal knowledge. His position is that we have a lot of ordinary modal knowledge concerning practical, scientific, and mathematical matters, but perhaps limited extraordinary modal knowledge. Extraordinary modal knowledge concerns matters on the periphery of scientific investigation or in the realm of metaphysical debate. He argues for his skepticism about extraordinary modal knowledge on the basis of an analogy with judgments of distance by the naked eye. He maintains that in a range of cases, naked-eye judgments of distance are reliable, though fallible; and likewise in a range of cases, modal judgments about ordinary practical matters and scientific matters are also reliable, though fallible. However, he argues that just as judgments of distance by the naked eye break down in certain cases, judgments about extraordinary modal claims based on conceiving or imagining a situation that appears to verify a statement equally break down. The main issue concerns how we can be confident that we have conceived things to the relevant level of depth required for the scenario to actually be a presentation or manifestation of a genuine possibility.

Given a particular statement S, van Inwagen raises the question: how does one know that the relevant depth of the scenario they have imagined is sufficient to ground the truth of the statement S? For example, conceiving of a situation in which mathematicians announce that a theorem has been proved is not sufficient for believing that the theorem is provable, since we can easily conceive of impossibilities being announced as proven by mathematicians. It would appear that what is required is for one to conceive of the proof itself or something in the vicinity of it that leads to a proof. With reference to the example of water, one might say that the reason one found the statement water is present without hydrogen conceivable is that one had not conceived of the scenario in sufficient enough detail. The appearance of possibility is explained by a failure to have the relevant depth of detail. Conceiving of a liquid and supposing that hydrogen is not a component of it does not constitute the relevant depth of detail. Much more would appear to be required, such as conceiving of how the liquid would still boil at its normal temperature without hydrogen. The general problem of conceiving to the relevant depth is exacerbated when our judgments concern extraordinary modal claims where we are perhaps less confident about what relevant details would need to be in place for a coherent scenario to reveal a genuine possibility rather than a mere appearance of possibility. For example, what grounds our confidence that we have conceived of a mind without a body simply by conceiving of consciousness without a body being present? For instance, one could imagine that someone is consciously thinking about something while just affirming abstractly that no body is present where the thinking occurs. But is that sufficient? Perhaps much more detail is required to verify that we have conceived of consciousness without materiality.

The challenge van Inwagen sets for modal epistemology is the following: how does one know (or how can one be confident) that one has reached sufficient detail in the scenario they have imagined so as to have included in it the truth of the claim in question rather than an unreliable sign of the truth? Geirsson (2005) and Hawke (2011) have further debated the issue discussed by van Inwagen.

4.3.3 The Causal Isolation Problem

One fundamental problem in the epistemology of modality stems from possible worlds semantics. Recall that (PWS), roughly, is the view that the truth conditions for

1. It is possible that P.
2. It is necessary that P.

are

3. P is true in some possible world.
4. P is true in all possible worlds.

The core idea is that possibility is truth in some world while necessity is truth in all worlds. The potential problem caused by possible worlds semantics is the causal isolation problem. The problem can be formulated as follows:

Realism:

Realism about possible worlds in the metaphysics of modality maintains that (i) facts about possible worlds are the truth-makers for modal statements, and (ii) that possible worlds are not causally connected to the actual world, either because a possible world is a comprehensive concrete universe that is causally isolated from our world or because a possible world is an abstract object, and in virtue of being an abstract object it has no causes or effects on the actual world.

Causal Condition:

X has knowledge of P only if X bears a causal connection to the truth-maker of P.

If one accepts Realism and Causal Condition, then there is a prima facie question: How can we ever know anything about metaphysical modality if we do not bear a causal connection to the truth-makers of modal statements?

The motivation for realism about modality comes from a commitment to the mind-independence of the truth-makers for modal claims. The core idea is that what makes a possibility or necessity claim true is not some fact about human minds, but some fact about the entities themselves. "It could have been the case that Rachel has a brother" is true not because Rachel can merely imagine it. Rather, it is true because something independent of her mind grounds the truth, in the case of (PWS), that independent something is part of a possible world.

The motivation for the causal condition often comes from an examination of cases of perception. When perception provides knowledge, part of the explanation appears to be that a causal connection obtains between the subject and the truth-maker of one's belief. For example, on some accounts of knowledge, seeing a fish in a bowl can provide one with knowledge of the fact that there is a fish in the bowl, partly in virtue of the fact that there is a causal relation that obtains between a fact in the world and the perceiver's mind.

It is important to note that the causal condition has been argued by some to be either categorically inappropriate or irrelevant as a requirement on a domain that is essentially non-spatio-temporally related to us. The general idea is that a causal condition is appropriate for concrete objects in the spatio-temporal realm, but not for entities outside of the spatio-temporal realm. For discussion of this issue see Lewis (1986). The problem as debated in the contemporary literature for the case of modality finds its most explicit expression in Peacocke's (1997) discussion of the integration challenge for modality, and his landmark (1999) work Being Known. For further discussion of Peacocke's solution see Roca-Royes (2010), and for critical discussion of how to eliminate the challenge see Bueno and Shalkowski (2004, 2014).

4.3.4 Skepticism based on Evolution

A related worry to the causal isolation problem comes from naturalistic accounts of epistemology that are grounded in the idea that our capacities for knowledge must be consistent with evolutionary explanations of our cognitive capacities. The arguments are aimed at the very possibility of having justification for beliefs about metaphysical modality. The problem is developed most directly by Nozick (2003: Ch. 3), and depends on two claims: (i) a necessary condition for being justified in believing that P is that a subject have a reliable belief forming module or faculty for the domain in question, and (ii) that evolution by natural selection provides the best explanation for which reliable belief forming mechanisms we possess. The Nozickian evolutionary skeptic argues as follows:

1. There is no adaptive advantage to getting things right about all possible worlds.
2. If there is no adaptive advantage to getting things right about all possible worlds, then there is no module or faculty for detecting truths about all possible worlds; and since truth in all possible worlds is the definition of metaphysical necessity, there is no module or faculty for detecting metaphysical necessity.
3. If there is no reliable module or faculty for detecting necessity, then none of our beliefs about necessity are justified.
4. So, we are not justified in any of our specific beliefs to the effect that something is metaphysically necessary.

There are three kinds of claims that the Nozickian skeptic brings forth to establish (1):

a. Our ability to imagine different scenarios is constrained by how evolution engineered our mind, and as a consequence it may not have the power to consider all the possible scenarios.
b. Whenever we have an appearance of possibility or necessity, the appearance is best explained as being about something other than metaphysical possibility or necessity.
c. There may be an adaptive advantage to having appearances of impossibility, when in actuality what appears impossible is possible.

Although (a)–(c) are controversial. Some initial plausibility can be given to each.

One reason to accept (a) is that there is no reason to believe that our imagination should be able to track all possibilities. It is likely that our imagination was engineered through evolution to deal primarily with local possibilities in our environment, such as the possibility of an object located in one place being located at another place or the possibility of an object moving at one speed moving at a much faster speed. In dealing with local possibilities, it may not have the capacity to consider all possibilities reliably.

One reason to accept (b) is that metaphysical possibility and necessity, as defined as truth in some possible world and truth in all possible worlds may itself reduce either to logical possibility and necessity or physical possibility and necessity. For our appearances of possibility and necessity to be about metaphysical possibility and necessity it must be the case that the best explanation is that there is a unique kind of modality picked out by "metaphysical modality" and that this modality is the best explanation for what our appearances of possibility and necessity are really about. If metaphysical modality collapses either into logical modality or physical modality, then there is no reason to believe that our appearances of possibility and necessity are really about metaphysical modality.

One reason to accept (c) is by analogy. Appearances of the world often present things to us in a way that may be better for us to process for the purpose of survival. Take the case of perception. On one account of perception and the world, the manifest image of the world as containing medium-sized objects, such as tables and trees, is false. Fundamental physics seems to be capable of complete explanations with no need for tables and trees, so perhaps they don't really exist.. However, it may be that for human survival it is better for us, in perception, such as vision, to see things as medium-sized dry goods, such as tables and trees, since it is easier for us to navigate and organize our lives around such macroscopic entities. In addition, it may be that there are certain possibilities that we cannot imagine simply because it is better for us either not to be able to see the possibility or because the forces that drove evolution pushed our minds to a place where taking something to be impossible was better than revealing it to be possible.

It is important to note that Nozick's argument depends on the claim that if there is no reliable module or faculty for detecting necessity, then none of our beliefs about necessity are justified. With respect to this assumption one might argue that although there is no specific faculty for detecting necessity, we are capable of reasoning our way to necessity by way of other faculties that we do have. Counterfactual theories of the epistemology of modality typically take this approach

4.4 RATIONALIST THEORIES

Rationalist theories, in one way or another, are grounded in the idea that despite the existence of a posteriori necessities, there is still a great deal of modal knowledge to be gained through a priori means. These views are often not concerned with modal knowledge with respect to a priori matters, such as in the case of logic and mathematics. Rather, these views are concerned with the extent to which we can have rational modal knowledge of matters outside of logic and mathematics, such as with respect to natural kinds or consciousness. The views differ on how much a priori knowledge they endorse, and how they account for it. In this section I review David Chalmers's Modal Rationalism, Christopher Peacocke's Principles of Possibility, E.J. Lowe's Serious Essentialism, and Bob Hale's Essentialism. Important rationalist accounts, not discussed here, are: Laurence Bonjour's (1998) In Defense of Pure Reason, George Bealer's (2002) The Rationalist Renaissance, Keith Hossack's (2007) The Metaphysics of Knowledge, Jonathan Ichikawa and Benjamin Jarvis's (2011) Rational Imagination and Modal Knowledge, and Christian Nimtz's (2012) Conceptual Truths, Strong Possibilities, and Metaphysical Necessity. In studying rationalist theories it is important to note that some theories may not give an explicit answer to the central question. Rather, they may give an account of what the connection is between the a priori and the necessary or between conceptual truths and necessity; or they may give an account of how intuition is reliable, and then argue that modal knowledge can be gained by way of intuition. The theories below are discussed because they aim to directly address the central question.

4.4.1 Modal Rationalism

In a series of papers (1996, 2002, 2010: Ch. 6) David Chalmers articulates, defends and responds to a number of objections to the view that conceivability entails possibility. Chalmers's account is not the only account of conceivability in the contemporary literature. Both Yablo (1993) and Menzies (1998) provide important accounts of conceivability. The main difference between their accounts and Chalmers's is that their views are defenses of evidential theories as opposed to entailment theories. An evidential account aims to show how conceivability provides evidence for possibility. An entailment account goes further and aims to show how in specific cases conceivability entails possibility. Evidential accounts face the problems posed by the existence of a posteriori necessities and the issue of conceiving to the relevant depth of detail. By contrast, Chalmers's Modal Rationalism is an entailment account; and thus must go beyond what evidential accounts offer. His main positive thesis is:

Weak Modal Rationalism (WMR):

Primary Positive Ideal Conceivability entails Primary Possibility.

(WMR) is constructed out of three distinctions:

i. Prima facie vs. Ideal rational reflection.
ii. Positive vs. Negative conceivability.
iii. Primary vs. Secondary conceivability/possibility.

The first distinction pertains to the issue of what kind of reasoning has gone into what one has conceived. A prima facie conception is just a person's initial reaction to a scenario, without reasoning further about the scenario. Better reasoning often gives one reason to doubt a prima facie conception. Ideal rational reasoning, by contrast, is reasoning that cannot be weakened by further reasoning. When an entailment link between conceivability and possibility is to be forged, the kind of reasoning involved has to be ideal. This distinction is used to deal with the problem of relevant-depth. At the level of ideal reasoning the relevant-depth of detail in the scenario has, arguably, been reached.

The second distinction pertains to two distinct ways in which one can engage in conceiving. Positive conceivability corresponds to actually constructing a scenario. In such a case one constructs a story in which a proposition can be verified to be true by the available details given. The story need not be a complete description of a scenario, but it must be sufficiently detailed so as to verify the statement being considered. By contrast, negative conceivability corresponds to not being able to rule out a certain statement. Negative conceivability is often weaker than positive conceivability, since it often derives from ignorance of the relevant facts. For example, if one does not know that water is identical to H_2O, they may find the statement "water does not contain hydrogen" conceivable because they cannot rule out the statement "water does not contain hydrogen" as being a priori incoherent. By contrast, conceiving of water without hydrogen in the positive sense requires constructing a scenario in which water is present without hydrogen at the relevant depth of detail required to verify the claim. Arguably, that sort of scenario cannot be constructed.

The third distinction pertains to two distinct ways in which we can evaluate statements across possible worlds. The distinction between primary and secondary conceivability/possibility rests on two independent theories: Epistemic Two-Dimensional Semantics (E2-D) and Modal Monism (MM). Each of these theories is at the heart of Chalmers's impressive contribution to the epistemology of modality. For an extended discussion of each see Chalmers (2004, 2010). For discussion of a related account of two-dimensional semantics see Jackson (1998, 2004). For an extended more complete discussion of Two-Dimensional Semantics see Schroeter (2012).

The distinction between primary and secondary conceivability and possibility is used to overcome the problem posed by the existence of a posteriori necessities in a way that allows for an entailment link between conceivability and possibility to be forged. What follows first is an intuitive account, followed by a brief technical account of Chalmers's modal rationalism.

Consider the question: Could water have been something other than H_2O? On (E2-D) there is both a yes answer and a no answer depending on how we read the question.

The yes answer comes from reading the question as follows: what would our term "water" have picked out, were we to have applied it to something that looks like water, but has a different chemical composition? That is, we can imagine a substance that looks like water, plays the actual world water-role, but in fact is some other chemical substance. And, we can imagine ourselves having used the term “water” to pick out that substance, rather than H_2O. The yes answer comes from thinking about what “water” would have picked out in a world where a different substance plays the water-role.

The no answer comes from reading the question as follows: given what water actually is, what could it have been? We used the term “water” to pick out a certain substance in our environment that plays a certain role. Scientists have discovered that water is identical to H_2O. We also have good reason to believe water is essentially H_2O. That is, we hold that water’s fundamental chemical nature reveals the essence of what water is. Now if we take the essentialist claim seriously, then we cannot imagine a world in which water is not H_2O because to imagine water is to imagine H_2O. The no answer comes from thinking about what variations water can undergo, given what we have discovered about its essence.

The intuitive explanation is rendered precise through the (E2-D) model that allows for the construction of an a priori link between conceivability and possibility by (i) making conceivability and possibility primarily a property of statements; (ii) distinguishing two kinds of intensions governing statements; (iii) acknowledging one space of worlds over which statements are evaluated; and (iv) distinguishing between two kinds of conceivability and possibility for statements corresponding to each of the intensions. Primary conceivability and possibility are then argued to allow for an entailment between conceivability and possibility.

The distinction between primary and secondary intensions has undergone several revisions and refinements since Chalmers (1996). It is a technical distinction. For the purposes of discussion and understanding, here, I will be presenting a brief formal account of the distinction with respect to the core problem posed by a posteriori necessities. Where S is a statement the distinction between primary and secondary intensions is the following:

1. The primary intension of S is a function from scenarios to truth-values. The primary intension of S is determined by asking an actual world evaluation question: If the scenario w turns out to be the actual world, what is the truth-value of S in w?
2. The secondary intension of S is a function from worlds to truth-values. The secondary intension of S is given by asking a counterfactual world evaluation question: Given that w is the actual world, what is the truth-value of S in a distinct world w*?

With the distinction in place the critical question is: how does the distinction between primary and secondary intensions ameliorate the problem posed by the existence of a posteriori necessities so as to enable an entailment between conceivability and possibility? To show how the distinction ameliorates the problem, consider the following example concerning the identity of Hesperus and Phosphorus. Assume, as it is actually the case, that:

a. “Hesperus” is a name of the planet Venus, it was introduced by the description H1H1 = the brightest star seen in the morning. The name “Hesperus” is a rigid designator (it picks out the same thing in all possible worlds where it has reference).

b. “Phosphorus” is a name of the planet Venus, it was introduced by the description P1P1 = the brightest star seen in the evening. The name “Phosphorus” is a rigid designator (it picks out the same thing in all possible worlds where it has reference).

c. It was an empirical discovery that Hesperus = Phosphorus.

d. It is metaphysically necessary that Hesperus = Phosphorus, since an identity statement between rigid designators captures a metaphysically necessary identity claim. In addition, this metaphysical necessity can only be known a posteriori, because Hesperus = Phosphorus is only knowable a posteriori.

Now suppose a thinker that knows that Hesperus = Phosphorus aims to conceive of a scenario SS in which Hesperus $\neq\neq$ Phosphorus in order to determine whether it is possible that Hesperus $\neq\neq$ Phosphorus. In constructing SS they imagine a scenario in which a planet takes one orbital path and another planet takes a distinct orbital path. Question: Is SS a situation in which one has conceived of Hesperus being non-identical to Phosphorus? According to Kripke the answer is no, because in SS one has simply conceived of a scenario in which our ordinary means of access to the referent of "Hesperus" and "Phosphorus" are occupied by distinct planets. These two planets cannot be Hesperus and Phosphorus, because Hesperus = Phosphorus necessarily.

By contrast, the story that weak modal rationalism offers is the following.

When constructing SS we have two options. We can either construct SS using the names "Hesperus" and "Phosphorus" or we can use the descriptions H1H1 and P1P1. If we use the names and take into consideration the fact that Hesperus = Phosphorus, then we must come to the conclusion, as Kripke does, that SS is not a situation in which Hesperus ≠≠ Phosphorus. However, if we use the descriptions H1H1 and P1P1 and ask ourselves the question "what in a given possible world answers to these descriptions?" we may find out that H1H1 and P1P1 are satisfied by two distinct planets. Why? Because it is not necessary that H1=P1H1=P1. There are possible worlds in which the brightest star seen in the morning is not identical to the brightest star seen in the evening. In short, the fact that "Hesperus = Phosphorus" is necessary and knowable only a posteriori does not block the a priori conceivability of "Hesperus ≠≠ Phosphorus" when we conceive of things only using H1H1 and P1P1, the descriptions we used to fix the reference of "Hesperus" and "Phosphorus" in the actual world. When we conceive of a scenario in which H1H1 and P1P1 are satisfied by two distinct planets, we have conceived of a scenario in which Hesperus ≠≠ Phosphorus. The idea is that conceiving with primary intensions requires that we ask the question:

could it have turned out that the brightest star seen in the morning is not the same star as the brightest one seen in the evening?

This question is distinct from the question:

given that Hesperus = Phosphorus, could it have turned out that Hesperus is not Phosphorus?

The former question concerns primary conceivability, the latter concerns secondary conceivability.

With the distinction between primary and secondary intensions in place, Chalmers argues that while primary conceivability does not entail secondary possibility because of a posteriori necessities, primary conceivability under the right circumstances—positive ideal rational reflection—entails primary possibility.

4.4.2 Critical Questions for Conceivability

Conceivability accounts face a set of general critical questions.

The Connection Question: How is conceivability connected to possibility? Given that modality is mind-independent and conceivability is mind-dependent, how are the two connected such that conceivability provides evidence of possibility? The question becomes clear when one draws a contrast with perception. Perception, such as vision, generally has a connection to the objects that one perceives. And it is through the causal connection that one can argue that perception provides one with justification for believing something about their environment. By contrast, if possible worlds are causally isolated from us, how does mind-dependent conceivability provide one with justification for believing that something is mind-independently possible?

The Dependence Question: Suppose that conceivability does provide justification for believing that something is possible. Does it succeed in doing so simply because one possesses a distinct kind of modal or non-modal knowledge that allows for conceivability to operate so as to produce justification? For example, does conceivability guide one to the belief that a round square is impossible simply because one knows what squares and circles are, and by examining their definition one can arrive safely at the conclusion that such objects are impossible? Similarly, does one simply find water in the absence of hydrogen possible because one either suppresses the knowledge that water contains hydrogen or one does not know that water does contain hydrogen? The dependence question is important because part of the epistemology of modality is concerned with the question of modal architecture/epistemic priority: what is the source of modal knowledge? Is conceivability an ultimate source of modal knowledge, or is it a derivative source of modal knowledge, dependent on another source, such as knowledge of essence and essential properties?

The Conditions Question: suppose that conceivability does provide justification for believing that something is possible. Does conceivability ever entail possibility? If it does, what are the conditions one must be in for conceivability to entail possibility? Do humans ever instantiate those conditions? For example, in the case of Chalmers's weak modal rationalism one might agree that conceivability entails possibility in the sense he defends, but question whether humans are ever in the position of ideal rational reflection. See Worley (2003) for discussion.

The Direction Question: There are two directions in which conceivability can be discussed.

- (CP)If PP is conceivable, then PP is possible.
- (INCP)If PP is inconceivable, then PP is impossible.

It is theoretically possible that the two theses are logically independent. And that one is more reliable than the other. For example, one could argue that inconceivability is a reliable guide to impossibility, while conceivability is a not a reliable guide to possibility.

The Relational Question: what are the relations between the epistemic domain of a priori and a posteriori knowledge and the metaphysical domain of necessary, essential, and contingent truths? That is, independently of human cognition, what relations obtain between the epistemological and the metaphysical categories?

4.4.3 The Principles of Possibility

Following the work of Benacerraf (1973) in the philosophy of mathematics, Christopher Peacocke (1997, 1999) develops an epistemology of modality aimed at solving the integration challenge for modality. In general, for a given domain of discourse DD the integration challenge for DD is the challenge of integrating the metaphysics/semantics of DD with an epistemology of DD that ratifies our knowledge of the domain. On the assumption that moderate realism, which maintains that modal truths are mind-independent, is true for modal claims, the integration challenge for modality is to reconcile the mind-independence of modal claims with an epistemology that shows how we can know modal claims even though human thinkers do not bear causal relations to the relevant truth-makers for modal truths. That is, Peacocke aims to solve the causal-isolation problem. He believes that the best way to solve the problem is to adopt moderate rationalism, which seeks to explain cases of a priori knowledge by appeal to the nature of the concepts that feature in contents that are known a priori. (Peacocke 2004: 199)

In pursuing moderate rationalism for modality Peacocke develops the Principles of Possibility account.

The central commitment of Peacocke's account is that for a subject to possess the concept of metaphysical modality is for that subject to have tacit knowledge of a specific set of Principles of Possibility that govern their understanding and evaluation of modal discourse. An individual thinker's tacit knowledge of the Principles of Possibility and the role these principles play in their modal discourse is modeled on the way in which principles of grammaticality govern how normal adult speakers understand and evaluate grammaticality in their native language. The analogy is as follows.

- **Grammaticality**
- i."Mary school went" is ungrammatical.
- ii."Jasvir drove her car" is grammatical.

XX understands, evaluates, and makes grammatical claims, such as (i) and (ii), because XX has tacit knowledge of Principles of Grammaticality G1…GnG1…Gn in virtue of which grammatical claims, such as (i) and (ii), are understood, evaluated and hold true.

- **Modality**
- iii.It is possible for the chair located by the wall to be located in the corner.
- iv.It is necessary that any specific human, such as Sheba, is a member of a biological kind.

XX understands, evaluates, and is capable of making modal claims, such as (iii) and (iv), because XX has tacit knowledge of Principles of Possibility in virtue of which modal claims, such as (iii) and (iv), are understood, evaluated, and hold true.

Some central claims of the theory are:

Notes

1. A native adult English speaker can be said to know English grammar, and reliably judge that a sentence SS of English is grammatical even though they are unable to state explicitly the rules of English grammar that render SS grammatical.
2. A plausible explanation of how a native speaker of a language can be credited with making reliable and knowledgeable claims about the grammaticality of sentences in their native language is in virtue of the fact that they tacitly draw on and know the very principles of grammar that render sentences of the language grammatical. These principles and rules of grammar are for the most part not explicitly expressible by the subject, but they are tacitly known.
3. Likewise, a person that possesses the concept of metaphysical modality tacitly knows a set of Principles of Possibility in virtue of which any given metaphysically modal judgment holds true.
4. The Principles of Possibility are the principles that the subject tacitly draws on in making, evaluating and understanding metaphysically modal judgments.
5. The Principles of Possibility are tacitly known, rather than explicitly known.

Much of Peacocke's project consists in articulating and defending the Principles of Possibility. (For critical discussion of the Principles of Possibility approach see the symposium on Being Known in Philosophy and Phenomenological Research 64(3).) In general, there are two main critical issues that surround the Principles of Possibility. On the one hand, there are issues about circularity. It appears that at several places the conception potentially opens itself up to a charge of circularity in virtue of using one kind of modality to explain another kind of modality. For example, genuine possibility is explained via admissibility of assignment. However, admissibility itself is a modal notion. Thus, one could question whether the modality involved in admissibility is problematic. Peacocke (1999) presents several responses to possible circularity objections. On the other hand, there are issues surrounding the kind of modality that is embraced by the approach. It appears that Peacocke's account acknowledges an actualist conception of modality rather than a possibilist conception. An actualist maintains that objects, properties and relations that actually exist constitute the basis for the construction of all possible worlds. A possibilist denies this, maintaining that in some possible worlds there are objects, properties, or relations that are not found in the actual world. One might worry that the principles articulated in the theory limit the approach to an actualist ontology. Peacocke (2002b) presents an extension of his view, which aims at accounting for some possibilist claims.

More recently, Sonia Roca-Royes (2010) draws attention to a distinct kind of circularity problem she calls the revenge of the integration challenge. The basic problem is that on Peacocke's epistemology of modality our knowledge of modality is parasitic on our knowledge of constitutive principles, whether these principles are implicitly or explicitly known. We determine that something is possible or necessary for an entity in part through our knowledge of what is constitutive of the entity. That is, what it is to be the kind of thing in question. For example, if we know that being human is a constitutive property of a given human, such as Tom, then we can come to know that it is impossible for Tom to be a zebra, but that it is possible for Tom to be born somewhat later than he was actually born. As a consequence of this relation between the role of constitutive principles and our evaluation of specific modal claims for the purposes of generating modal knowledge, a comprehensive account of modal knowledge is incomplete without a picture of how we come to know the relevant constitutive principles involved in our evaluations of modal knowledge. Thus, the integration challenge returns when we ask the question: how do we arrive at our knowledge, implicit or explicit, of the constitutive principles that play a role in explaining our modal knowledge? This question is important because arguably in the case of grammaticality there is an innate universal grammar that aids in the acquisition of a local grammar, such as English; by contrast, in the case of modality it could be that no innate universal modal principles exist. Peacocke himself notes the worry,

the provision of a general theory of the constitutive, as opposed to the modal, seems to me to be an urgent task for philosophy. We certainly do not want all the initial puzzlement about modality simply to be transferred to the domain of the constitutive. Only a satisfactory general theory of the constitutive, and an attendant epistemology, can allay this concern. (Peacocke 1999: 166, fn.37)

4.4.4 Essentialist Deduction

E.J. Lowe (2008a, 2012) and Bob Hale (2013) have independently developed accounts of the epistemology of modality based on metaphysical essentialism. The two core theses of metaphysical essentialism are: (i) entities have essential properties or essences that are not merely dependent on language, and (ii) not all necessary truths capture an essential truth or the essence of an entity. Although their views differ at crucial points in the epistemic landscape, the program they share maintains the following:

Metaphysical Grounding:

The essential properties or essences of entities are the metaphysical ground of metaphysical modality. When we look for an explanation of why something is metaphysically possible or necessary we ultimately look to the essential properties or essences of the entities involved.

Epistemic Guide:

The fundamental pathway to acquiring knowledge of metaphysical modality derives from knowledge of essential properties or essences of the entities involved. When we look for an explanation of how we can know metaphysical modality we ultimately look to our knowledge of essential properties or essences as the basis upon which we make inferences to metaphysical modality.

As a general point, it is important to note that both Lowe and Hale can be taken to endorse symmetric essentialism, which is the view that essence is both the ground and the epistemic pathway to modal knowledge. This view is to be contrasted with asymmetric essentialism, which holds that while essence is the ground of modality, it is not the epistemic pathway. An asymmetric essentialist holds that our knowledge of necessity is prior to our knowledge of essence. And that it is through a special investigation of necessities that we come to possess knowledge of essence by modal sorting.

From a metaphysical point of view both Lowe and Hale share the view that the essential properties of an entity are distinct from the mere metaphysical necessities that are true of the entity. This position is inspired by the work of Fine (1994) on the relation between essence and metaphysical modality. Fine argues against modal conceptions of essence on which it is claimed that an essential property of an object is simply any property the object has in all possible worlds in which it exists. He offers the following argument against the view:

i. Socrates is not essentially a member of the set only containing Socrates, abbreviated as: {Socrates}. It is not part of the essence of Socrates that he is a member of {Socrates}. What Socrates fundamentally is does not include being a member of {Socrates} through his real nature. Socrates's real nature is that of being a human. Being human, by itself, has no relation automatically to being the only member of a certain kind of set.

ii. In every possible world in which Socrates exists, sets also exist, since mathematical entities exist in all possible worlds. Thus {Socrates} exists in every possible world in which Socrates exists. As a consequence, Socrates has the property of being a member of {Socrates} in every possible world in which Socrates exists.

iii. It is false that EE is an essential property of xx if and only if xx has EE in every possible world in which xx exists.

Simply put, essential properties are more fine-grained than necessary properties. As a consequence, we cannot simply take essential properties or essences to be what an object has in every possible world in which it exists.

From an epistemological point of view both Lowe and Hale provide a picture of our knowledge of modality that sharply contrasts with accounts that take conceivability or intuition to be our fundamental source of justification for believing metaphysically modal truths. The core contrast, for example with conceivability, is that modal knowledge derives from essentialist knowledge, and that conceivability is explained as being successful only in virtue of our possession of essentialist knowledge that is unpacked in a conceivability exercise.

For the purposes of clarifying his approach, Lowe explains our knowledge of metaphysical necessities through the following procedure:

i. First, we arrive at a real definition of the entities in question, such as ellipses and cones, or statues and lumps of clay. A real definition of an entity or kind of entity either specifies what the entity is or what the kind is. This can be done either through a standard definition of the thing, or through a generating principle. Next, from an understanding of the relevant real definitions of the entities in question, we arrive at an understanding of their essential properties or essences, such as the essence of an ellipse, a cone, a statue, or a lump of clay.
ii. Second, we reason our way to a conclusion about what is compatible or incompatible with the relevant essential properties or essences.
iii. Third, using a principle linking essential properties and essences with metaphysical necessity and possibility, we conclude that a certain proposition, derived from claims involving the essential properties or essences of the relevant entities in question, is metaphysically necessary or possible.

Both Lowe and Hale offer an account that aims to validate the following pattern of inference:

1. The real definition of XXs is CC.

therefore

2. The essence of XXs is CC.
3. If the essence of XXs is CC, and RR is a property incompatible with CC, then it is metaphysically impossible for XXs to have property RR.
4. CC is incompatible with RR.

therefore

5. It is metaphysically impossible for XXs to have property RR.

For example, the real definition of a circle is that it is a set of points in a plane equidistant from a given point. As a consequence, the essence of a circle is that a circle is a set of points in a plane equidistant from a given point. The property of being (a circle) an entity that is a set of points in a plane equidistant from a given point is incompatible with the property of being (a rectangle) a four-sided closed figure consisting of four right angles. Thus, given the essence of circles, it is metaphysically impossible for a circle to have the property that defines rectangles.

4.4.5 Critical Questions for Essentialism

Essentialism faces a set of critical questions.

i. What is the fundamental epistemic relation that essentialism is based on? Is it knowledge of essence, justification for beliefs about essence, or understanding of essence that is the basic epistemic relation?

ii. What is the essence of an entity? Are essences the sum of their essential properties? Are essences distinct existences from those things that they are essences of?

iii. What is an essential property, in addition to being a property that an entity has in every possible world where it exits?

iv. Given that there are mathematical kinds, such as circles and numbers, natural kinds, such as water and lightning, and social kinds, such as chairs and paintings, how is it that we can come to know the essence of these distinct kinds of things? Is it the same in all of these cases?

v. Do all entities have exactly the same kind of essence? Do social kinds have the same kind of real nature or essence that natural kinds and mathematical kinds possess?

vi. For every entity or kind of entity are its essential properties or essence known a priori or are some known a posteriori?

vii. How is the connection or bridge principle between essence and modality known?

These questions allow for a critical examination of essentialist type accounts. For example, concerning (i), Vaidya (2010) defends an understanding-based account of essence, while Lowe and Hale defend a knowledge-based, or what is known as an essentialist-k style theory. Concerning (vii), Horvath (2014) has argued that Lowe’s account of essentialist-k theory suffers from a prima facie problem. An outline of the problem is as follows:

Notes

1. For SS to know that it is possible for ee to be FF, SS must know: (i) either some essential properties of ee or the essence of ee, and (ii) the bridge principle (B), that if HH is the essence of ee, and FF is incompatible with HH, then it is impossible for ee to be FF.
2. Assume that SS possesses essentialist knowledge concerning ee. Question: how can SS know (B)?
3. (B) can be known either through (i) intuition, (ii) conceptual analysis, (iii) conceivability, or (iv) via counterfactual imaginability.
4. Lowe denies that (i)–(iv) are valid ways of knowing in the epistemology of modality. (Lowe 2012: Section 1)
5. Lowe argues for the no-further-entity account of essence on which an essential property or an essence of an entity is no further entity over and above the entity it is an essence of.
6. Given (3)–(5), one can argue that it is unlikely that Lowe can provide an account of our knowledge of possibility on the basis of our knowledge of essence.

The core problem is that by saying there is a single source for modal knowledge—via knowledge of essence—Lowe has potentially undermined his ability to provide an account of how one can know (B). One route that is plausible is the following. Argue that (i) conceptual analysis is how we come to know (B), (ii) in all cases of modal knowledge we reason by way of essence, and (iii) as a consequence the epistemology of modality is non-uniform. However, Lowe cannot adopt this route, since he has ruled out knowledge of modality by (i)–(iv). In contrast to Lowe's account, it is possible for Hale to offer an account of (B) through the use of conceptual analysis or through a treatment of the real definitions of essence and metaphysical modality.

Finally, one important issue that separates Lowe's account from Hale's is Lowe's commitment to epistemic essentialism, which Hale does not endorse. Lowe articulates his epistemic essentialism in his (2008a).

[E]ssence precedes existence. And by this I mean that the former precedes the latter both ontologically and epistemically. That is to say, on the one hand, I mean that it is a precondition of something's existing that its essence—along with the essences of other existing things—does not preclude its existence. And, on the other hand … I mean that we can in general know the essence of something XX antecedently to knowing whether or not XX exists. Otherwise, it seems to me, we could never find out that something exists. For how could we find out that something, XX, exists before knowing what XX is—before knowing, that is, what it is whose existence we have supposedly discovered? (Lowe 2008a: 40)

The epistemic position can be properly captured as:

Epistemic Essentialism:

knowledge of essence must precede knowledge of existence.

And it can be contrasted with two distinct views.

Epistemic Existentialism:

knowledge of existence must precede knowledge of essence.

Epistemic Entanglement:

knowledge of essence neither necessarily precedes knowledge of essence nor is necessarily preceded by knowledge of existence.

4.5 COUNTERFACTUAL THEORIES

4.5.1 Counterfactuals and Modal Knowledge

Williamson (2005, 2007a,b), Hill (2006), Kroedel (2012), and Kment (2014) have all offered counterfactual theories of modal knowledge. While the four accounts share formal similarities, in this section the focus will be on Williamson's account. He partially describes his project in the epistemology of metaphysical modality through discussion of the philosophy of philosophy.

Humans evolved under no pressure to do philosophy. Presumably, survival and reproduction in the Stone Age depended little on philosophical prowess, dialectical skill being no more effective then than now as a seduction technique and in any case dependent on a hearer already equipped to recognize it. Any cognitive capacity we have for philosophy is a more or less accidental byproduct of other developments. Nor are psychological dispositions that are non-cognitive outside philosophy likely suddenly to become cognitive within it. We should expect cognitive capacities used in philosophy to be cases of general cognitive capacities used in ordinary life, perhaps trained, developed, and systematically applied in various special ways, just as the cognitive capacities that we use in mathematics and natural science are rooted in more primitive cognitive capacities to perceive, imagine, correlate, reason, discuss… In particular, a plausible non-skeptical epistemology of metaphysical modality should subsume our capacity to discriminate metaphysical possibilities from metaphysical impossibilities under more general cognitive capacities used in general life. I will argue that the ordinary cognitive capacity to handle counterfactual carries with it the cognitive capacity to handle metaphysical modality. (2007b: 136)

Williamson's counterfactual theory allows for the construction of an abductive anti-skeptical argument against Nozick's (2003) evolutionary-based skepticism about our knowledge of metaphysical modality.

1. Skepticism about knowledge of counterfactual conditionals is implausible, since knowledge of counterfactuals is pervasive for human decision-making, planning, and theory construction.
2. Metaphysical possibility and necessity are logically equivalent to counterfactual conditionals.
3. Skepticism about knowledge of metaphysical modality independently of skepticism about counterfactual conditionals is uneconomical and implausible, given that the capacity to handle counterfactuals in reasoning brings along with it the capacity to handle metaphysical modality.

therefore

4. Skepticism about knowledge of metaphysical possibility and necessity is implausible.

The key theses of Williamson's counterfactual theory are:

Logical Equivalence:

metaphysical possibility and necessity can be proven to be logically equivalent to counterfactual conditionals.

Epistemic Pathway:

counterfactual reasoning in imagination through the method of counterfactual development can provide one with justified beliefs or knowledge about metaphysical possibility and necessity.

Williamson presents his proof of the logical equivalence between counterfactuals and metaphysical modality by engaging the work of Robert Stalnaker and David Lewis. However, he does not commit himself to any specific account of the truth-conditions for counterfactual conditionals. The basic idea he employs from Stalnaker and Lewis is the following:

Where “A>BA>B” express “If it were that AA, it would be that BB”, (CC) gives the truth conditions for subjunctive conditionals: A subjunctive conditional “A>CA>C” is true at a possible world ww just in case either (i) AA is true at no possible world or (ii) some possible world at which both AA and CC are true is more similar to ww than any possible world at which both AA and ¬C¬C are true.

With (CC) and “⊥” as a symbol that stands for contradiction, Williamson proves the following logical equivalences between counterfactuals and metaphysical modality:

- (NEC)□A☐A if and only if (¬A>⊥)(¬A>⊥)

It is necessary that AA if and only if were ¬A¬A true, a contradiction would follow.

- (POS)◊A◊A if and only if ¬(A>⊥)¬(A>⊥)

It is possible that AA if and only if it is not the case that were AA true, a contradiction would follow.

The basic epistemic idea is that a justified belief about necessity and possibility can be arrived at through a counterfactual development, in imagination, of the supposition that ¬A¬A, for the case of necessity, and the supposition that AA, for the case of possibility.

Consider the following example from Williamson.

Suppose that you are in the mountains. As the sun melts the ice, rocks embedded in it are loosened and crash down the slope. You notice one rock slide into a bush. You wonder where it would have ended if the bush had not been there. A natural way to answer the question is by visualizing the rock sliding without the bush there, then bouncing down the slope into the lake at the bottom. Under suitable background conditions, you thereby come to know the counterfactual:

- (*)If the bush had not been there, the rock would have ended in the lake.

(2007b: 142)

According to his theory the general procedure we use to arrive at (*) is the following:

[O]ne supposes the antecedent and develops the supposition, adding further judgments within the supposition by reasoning, offline predictive mechanisms, and other offline judgments. The imagining may but need not be perceptual imagining. All of one's background beliefs are available from within the scope of the supposition as a description of one's actual circumstances for the purposes of comparison with the counterfactual circumstances… Some but not all of one's background knowledge and beliefs are also available within the scope of the supposition as a description of the counterfactual circumstances, according to complex criteria… To a first approximation: one asserts the counterfactual conditional if and only if the development [of the antecedent] eventually leads one to add the consequent. (2007b: 152–153)

From (*) and (POS), one can reason their way to the modal claim (**) by checking whether the development of the counterfactual yields a contradiction.

- (**)It is possible for the rock to have ended in the lake.

The counterfactual theory, thus, holds the following.

In the case of necessity: if a robust and good counterfactual development of ¬A¬A yields a contradiction, we are justified in asserting that AA is necessary. And, if a robust and good counterfactual development of ¬A¬A does not yield a contradiction, we are justified in denying that AA is necessary.

In the case of possibility: we are justified in asserting that AA is possible when a robust and good counterfactual development of the supposition that AA does not yield a contradiction. And we are justified in denying that AA is possible when a robust and good counterfactual development of AA yields a contradiction.

An important component of Williamson's account derives from his commentary on the traditional distinction between a priori and a posteriori knowledge. Contemporary theorists often maintain that what separates the a priori from the a posteriori is that in the former case experience only plays an enabling role—a role in enabling possession of a concept for an individual thinker—while in the latter case experience plays not only an enabling role, but an evidential role—the justification for a claim involving the concept requires appeal to experience by the thinker making the claim. Williamson maintains that several instances of counterfactual knowledge (the route by which we acquire modal knowledge) will be neither a priori nor a posteriori in any deep or insightful sense. Rather, he acknowledges an extensive category of armchair knowledge under which many cases of our knowledge of metaphysical modality would fall.

We may acknowledge an extensive category of armchair knowledge, in the sense of knowledge in which experience plays no strictly evidential role, while remembering that such knowledge may not fit the stereotype of the a priori, because the contribution of experience was far more than enabling. (2007b: 169)

He defines armchair knowledge as knowledge that is either strictly a priori knowledge or not strictly a priori or a posteriori. In the latter case, the knowledge is such that experience plays no strictly evidential role, but at the same time the role of experience does not fit the model of a priori knowledge, since far too much experience played a role in enabling concept possession and reliable use. Given Williamson's acknowledgement of armchair knowledge as a domain into which many instances of modal knowledge fall, it is best to describe his view as being an armchair account of modal knowledge, as opposed to a strictly rationalist or non-rationalist account.

4.5.2 Critical Questions for Counterfactual Imaginability

There are at least four kinds of critical questions that one can ask about counterfactual imaginability as a theory of our knowledge of metaphysical modality.

The Question of Dependence: Does the counterfactual account of our knowledge of metaphysical modality depend on any kind of modal knowledge? If so, is that dependence problematic? Williamson argues that we can come to possess modal knowledge, such as that it is possible for a rock located at LL to be located nearby at L∗L∗. This knowledge can be arrived at through counterfactual reasoning in imagination. However, one might ask does this counterfactual reasoning depend itself on any kind of modal knowledge or essentialist knowledge? Does one need to know in some problematic sense what essentially a rock is or what is possible for a rock, for one to reason counterfactually and correctly to the conclusion that a rock located at LL could be at L∗L∗ without contradiction?

The Question of Imaginative Engagement: Since the counterfactual account of our knowledge of metaphysical modality depends on counterfactual reasoning in imagination, what are the details of how the counterfactual imagination works? What can we learn about the conditions under which the counterfactual imagination is fallible or likely to be successful? What guides our counterfactual development? Why are we prone to imagine things unfolding in one manner rather than another? For example, when we generally imagine where a rock would have landed had a bush not been in its path, we don't typically imagine that the rock would have suddenly reversed direction from its current path. More over: what epistemic relevance does the fact that our imagination takes certain directions rather than others have on the epistemic status of our counterfactual development of a subjunctive conditional?

The Question of Scope: Given that the counterfactual account of our knowledge of metaphysical modality aims to capture metaphysical modality, does it really do so for the wide range of metaphysically modal claims that are known? Ordinary modal claims, such as that a bush located at LL, could be located at L∗L∗, appear to be non-problematic for the very reasons Williamson offers. However, can the account also provide us with modal knowledge of extraordinary modal claims, such as that it is possible for there to be a physical duplicate of a human that is not conscious? If the theory can only deliver knowledge of ordinary, as opposed to extraordinary, modal knowledge, is this a problem?

The Question of Adequacy: Williamson's account aims to explain our knowledge of modality via our general capacity to handle counterfactuals. One critical question is whether the strategy is explanatorily adequate. For example, Malmgren (2011: 307) questions

Williamson's assumption that we do have a general capacity to handle counterfactuals:

Is it legitimate to suppose that we do have a general capacity to handle counterfactuals? I will argue that it is not; more precisely, that it is not legitimate to suppose that we have a general capacity at the appropriate level of implementation.

Malmgren's argument aims to show that even though there might be good reasons to reject rationalism about knowledge of metaphysical possibility, Williamson's argument against rationalism fails. The core of her argument is as follows:

1. Let rationalism be the view that our knowledge of metaphysical modality is a priori and that we possess a special faculty for acquiring knowledge of metaphysical modality.
2. The counterfactual theory of modal knowledge that Williamson defends can be seen to be an attempt to explain modal knowledge in terms of counterfactual knowledge so that there is no need to posit a special faculty that provides us with a priori justification for knowledge of metaphysical modality. The counterfactual theory provides an armchair account of our knowledge of metaphysical modality, and not a strictly rationalist account.
3. However, the appeal to the logical equivalence between counterfactuals and metaphysical modality does not show that there is no special-faculty for reasoning about metaphysical modality at a lower level of implementation that is a priori across a range of philosophically interesting cases involving metaphysical modality.

therefore

4. The argument against rationalism, via the appeal to our general capacity to handle counterfactuals, fails.

The core of Malmgren's argument rests on (3). She offers several reasons, which are paraphrased below.

i. There is a trivial and uncontroversial sense in which we have the capacity to handle counterfactuals. This trivial and uncontroversial sense does not compete with rationalist explanations of our knowledge of metaphysical modality at the same level of explanation. (2011: 309)

ii. A general capacity to handle counterfactuals can be implemented in distinct ways even within the same subject. (2011: 309–310)

iii. If there are multiple mechanisms and ways in which our general capacity to handle counterfactuals can be realized, then it is theoretically possible that in the case of metaphysical modality there is a more specific mechanism at play, and that it provides a priori justification over a range of philosophically interesting cases, such as whether it is metaphysically possible for a person to have a justified true belief without knowledge. (2011: 310)

iv. Most cognitive scientists working on the evaluation of counterfactuals agree that counterfactual evaluation is far from a unified affair—it involves many different capacities and/or mechanisms. Which mechanism gets recruited in a specific case appears to depend, among other things, on the content and complexity of the given counterfactual claim, and the background beliefs of the subject. (2011: 311)

v. Counterfactual judgments are heterogeneous in the following respects. Some judgments are capable of being justified a priori and others are capable of only being justified a posteriori. For example: "If I had made the supper it would have been inedible" can only be justified a posteriori, while "If twelve people had been killed more than eleven people would have been killed" can be justified a priori. (2011: 315).

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What do you know about the Kripke on a posteriori Necessities and The Deduction Model?

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2. Discuss about the Epistemic Issues Pertaining to Kripke's Work.

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3. Describe Rationalist Theories.

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4. Describe Counterfactual Theories.

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4.6 LET US SUM UP

Three important questions for the theory are the following:

a. What specific details of relevant similarity does one need to know to be in a position to make the relevant inference? For example, does one need to simply know that Messy and Twin-Messy are the same kind of IKEA table? Could they know something less specific, such as that they are both wooden tables of roughly the same structure? Or do they need to know something more specific, such as that they are the same IKEA table from the same year and model of design?

b. How does the theory account for knowledge of possibility across distinct types of entities? For example, because Twin-Messy and Messy are the same type of IKEA table, it is reasonable to hold that knowledge of the fact that Twin-Messy broke can inform our knowledge of the breakability of Messy. However, suppose one has never owned a table before. Rather, they have only owned a bench before, and they have seen the bench break. Can knowledge of one type of entities modal characteristics provide us with grounds for knowledge of possibility for another type of entity?

c. How does knowledge of similarity allow us to gain knowledge of necessity? The account provided illustrates how prior knowledge of actuality can allow us to access knowledge of possibility. But we also know necessary truths: how do we arrive at knowledge of necessity?

4.7 KEY WORDS

Knowledge: Knowledge is a familiarity, awareness, or understanding of someone or something, such as facts, information, descriptions, or skills, which is acquired through experience or education by perceiving, discovering, or learning. Knowledge can refer to a theoretical or practical understanding of a subject.

4.8 QUESTIONS FOR REVIEW

1. Discuss the Problem of a posteriori Necessities.
2. What is the Relevant-Depth Problem?
3. What is Causal Isolation Problem?
4. What is Skepticism based on Evolution?

4.9 SUGGESTED READINGS AND REFERENCES

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4.10 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 4.2
2. See Section 4.3
3. See Section 4.4
4. See Section 4.5

UNIT 5: MODAL SYLLOGISMS

STRUCTURE

5.0 OBJECTIVES

In this unit an attempt is made:

• to introduce to you salient features of syllogism, which forms an important part of classical or Aristotelian Syllogism.

• to integrate traditional analysis with modern analysis. In doing so, some vital differences between these analyses are brought to the fore.

5.1 INTRODUCTION

Syllogism is the most important part of Aristotle's logic. It is a kind of mediate inference in which conclusion follows from two premises. We consider two kinds of syllogism, viz., conditional and unconditional. Further, under conditional, there are two divisions: mixed and pure. We can consider conditional syllogism at a later stage. In this unit, we shall confine ourselves to unconditional syllogism or categorical syllogism.

In this unit which makes definite advances in the understanding of Aristotle's modal syllogistic and of his non-modal syllogistic as well. It is very elegant, relative to the complexity of the material. Thanks to the carefully structured prose discussion in front and the collection of tables for reference in the back, a reader can gain a systematic grasp of Aristotle's commitments, the relations among them, and the prospects for a coherent interpretation of what Aristotle was up to. Moreover, the book develops a novel semantics for the sentences treated by Aristotle's modal syllogistic, according to which surprisingly many of Aristotle's claims come out true. This book deserves careful study by anyone with an interest in the history of logic. For researchers on Aristotle's Analytics, it will be an indispensable resource.

Aristotle's modal syllogistic, contained in Prior Analytics chapters 1.3 and 8-22, lies at the intersection of two of his great philosophical legacies. It belongs to the Prior Analytics, which is the founding text of formal logic. And it systematically considers how to reason about necessity and possibility, a question which Aristotle was the first thinker to tackle seriously and which he treated from different angles in different texts. Thus the modal syllogistic stands in connection with the assertoric syllogistic on the one hand (Pr. An. 1.1-2 and 4-7), and with various texts about necessity, possibility, and capacities on the other (for example, Metaph. IX). It is an extremely difficult text. If we can make sense of it, it should yield valuable information about Aristotle's conception(s) of modality and about his approach(es) to language and logic.

Chapters 1.3 and 8-22 of the Prior Analytics are messy and complicated. There are mistakes and contradictions, which no interpretation can get around, short of introducing grave ambiguities and equivocations into the text. (If I am not mistaken, Aristotle has contradicted himself by the end of chapter 1.9, scarcely halfway through his discussion of inferences involving necessity-sentences.[1])

Nevertheless, Marko Malink shows how to mine a consistent theory out of these chapters. He identifies a set of claims that are (a) delimited in a principled way and (b) plausibly regarded as the meat of Aristotle's modal syllogistic. In Malink's words, these are 'all of Aristotle's claims about the validity and invalidity of inferences in the modal syllogistic' (p. 2). I will say more in a moment about which claims are included and excluded by this description (the description may be slightly misleading).

Malink shows that this limited set of claims is consistent. He does this by providing an interpretation of their subject matter on which all the claims are true. More precisely, he provides two such interpretations: a weaker one in the appendix and a stronger one in the main text.

An example of a difference between Malink's two interpretations is that under the stronger interpretation, Barbari XNN is valid, while under the weaker one it is invalid.[2] Aristotle himself does not seem to take a view about Barbari XNN, which has the form 'A applies to all B; B necessarily applies to all C; therefore A necessarily applies to some C'.

Which exactly of Aristotle's claims is Malink's interpretation designed to make true?

The modal syllogistic is a theory about entailment relations among sixteen types of sentence. The sentences in question each contain two terms, one of which is said to apply (or not apply) to the other in some way. The sixteen types of sentence result from different combinations of quantity (universal or particular), quality (affirmative or negative), and modality (necessity, possibility [to the exclusion of necessity], possibility [perhaps including necessity], or absence of modal qualification). Examples: 'swift applies to all horses', 'asleep possibly applies to some horse', 'swan necessarily applies to no horse.' In the course of the chapters on modal syllogisms, Aristotle says various sorts of things about these sentences. These include:

Elucidations: Remarks about what a given type of sentence means. These are rare and not very informative.

Examples: Examples of concrete terms that yield a true, or false, sentence of a given type.

Conversion claims: Claims to the effect that a given sentence entails a corresponding sentence with the terms exchanged. For example, he claims:

'A necessarily applies to no B' entails 'B necessarily applies to no A'.

Claims of validity and invalidity: Claims to the effect that a given pair of sentences (the premises) entails, or does not entail, a third sentence (the putative conclusion). These include:

A) Syllogistic-style arguments: In most cases, the premises and putative conclusion conform to a certain pattern — one of Aristotle's 'three figures' — in which the two premises have a term in common and the putative conclusion contains the remaining two terms. For example, Aristotle claims:

'A necessarily applies to no B' and

'B applies to all C' entail:

'A necessarily applies to no C'

B) Nonsyllogistic-style arguments: In the course of discussing syllogistic-style arguments, Aristotle sometimes makes claims about sentence triplets that do not conform to the syllogistic pattern. For example, he claims:

'A applies to all B' and

'B necessarily applies to all C' do not entail:

'A necessarily applies to some B'.

This is not a syllogistic-style argument, because the putative conclusion contains the shared term B rather than containing both non-shared terms, A and C.

Malink's interpretation is designed to validate Aristotle's claims of validity and invalidity of syllogistic-style arguments, as well as his conversion claims.

The remaining sorts of claims are allowed to fall out as they may. Thus, not all of Aristotle's examples turn out correct: on some occasions, Aristotle claims that a given pair of terms yields a true (false) sentence of a given type although, under Malink's interpretation, the sentence in question is false (true). Similarly, some of Aristotle's claims of invalidity of nonsyllogistic-style arguments come out false. For example, under Malink's interpretation, 'A applies to all B' and 'B necessarily applies to all C' entail 'A necessarily applies to some B', contrary to what Aristotle says. (There is good reason for this; see note 1.)

In Malink's interpretations, the meaning of a sentence in Aristotle's modal syllogistic does not result in any straightforward, compositional way from the meanings of constituents such as 'necessarily', 'some', and 'not'. Instead, Malink's approach is holistic: the sixteen types of sentence contain sixteen different complex copulae, and each copula is given an interpretation as a whole package.

The interpretations are built out of three primitive relations between terms, called ‘a_X-predication’, ‘a_N-predication’, and ‘strong a_N-predication’. These three relations are governed by a handful of simple axioms (ax_1-ax_6, p. 287) and, in the main-text version of Malink’s interpretations, a number of additional theses as well (S1-25, pp. 116-159).

Sentences are interpreted in terms of more or less complicated constructions out of the three primitive relations. Starting with the simplest cases, ‘A applies to all B’ is true iff A is a_X-predicated of B, and ‘A necessarily applies to all B’ is true iff A is a_N-predicated of B.

‘A necessarily holds of some B’ is true iff either A is a_N-predicated of something of which B is a_X-predicated, or A is a_X-predicated of something of which B is a_N-predicated.

Other cases are increasingly complex.

For several types of sentence, especially those involving possibility, Malink acknowledges that his interpretations are artificial. They do not represent what the sentences in question really mean in the context of Aristotle’s theory, and they do not fully explain why Aristotle made the claims of validity and invalidity that he made. Still, Malink is right to insist on the usefulness of his interpretations. Most obviously, they serve to show that Aristotle’s claims are consistent. Moreover, they may explain some aspects of Aristotle’s thinking about the sentences in question without explaining all aspects. And they can provide the basis for a future compromise. We could give simpler and more natural interpretations to the sentences of Aristotle’s modal syllogistic, but, as a cost, we would have to write off some of his validity- and invalidity-claims as mistakes. Before settling on such a compromise, it is extremely useful to see, in detail, exactly what an uncompromising interpretation looks like.

For other parts of the modal syllogistic, Malink defends his interpretations as true accounts of the meanings of the sentences that Aristotle’s theory is about. Malink’s defense employs ideas about terms and predication, which he extracts from the Topics. The treatment of the Topics is very interesting and provocative. It should be read cautiously: perhaps everything Malink says is right, but at present I am unsure. Regardless, it is an enlightening read. Malink has reasons for everything he says, and when one disagrees, the disagreement is sure to be productive.

Malink finds in the Topics a distinction between three types of term. There are two types of what he calls essence terms: namely, substance terms (such as ‘Socrates’, ‘animal’, and ‘horse’) and nonsubstance essence terms (such as ‘color’ and ‘whiteness’: roughly, names of attributes). Essence terms come in genus-species trees: for example, animal is a genus of horse, and color is a genus of whiteness. Third, there are what Malink calls nonessence terms, such as ‘colored’ and ‘white’: roughly, adjectives. Nonessence terms, unlike their nominalized correlates, do not stand in genus-species relations. (Color is a genus of whiteness, but colored is not a genus of white.)

Malink maintains two key theses about these types of terms. The first thesis is that essence terms apply necessarily to anything they apply to. Part of the idea is that an essence term, such as ‘redness’, cannot be used to say that an attribute inheres in a subject. Only the correlated nonessence terms fulfill that function. For example, the fact that all fire hydrants are red is expressed by the sentence ‘red applies to all fire hydrants’, and not by the sentence ‘redness applies to all fire hydrants’. In this sort of way, essence terms are barred from serving as predicates in true contingent predicative statements. They are confined to such truths as ‘redness applies to all (shades of) scarlet’, cases where the corresponding necessity-sentence (e.g., ‘redness necessarily applies to all [shades of] scarlet’) is also true.

The second thesis is that whenever ‘A necessarily applies to all B’ is true, B is an essence term. The thought is that necessity is grounded in essence in such a way that a subject of necessary universal predication must have an essence.

Taken together, these two theses can explain several of Aristotle’s claims about inferences involving necessity-sentences. Most centrally, they yield an explanation of Aristotle’s endorsement of Barbara NXN, that is, his claim that

‘A necessarily applies to all B’ and
‘B applies to all C’ entail
‘A necessarily applies to all C’.

Malink’s explanation goes like this. Given that A necessarily applies to all B, the second thesis implies that B is an essence term. Given that B is an essence term and that B applies to all C, the first thesis implies that B necessarily applies to all C. (NB: ‘B necessarily applies to all C’ is not a further assumption or ‘shadow premise’ in Malink’s explanation. It is a consequence of the two original premises along with general theses about essence terms and nonessence terms.) Now we have ‘A necessarily applies to all B’ and ‘B necessarily applies to all C’, and it is plausible to hold that these entail ‘A necessarily applies to all C’.

Malink does an ingenious job at finding evidence for his explanation in Aristotle’s Topics. On the other hand, his account is liable to objections. There is not space to weigh the evidence here. I am uncertain whether Malink’s readings of the Topics are all ultimately justified, and whether his explanation of Barbara NXN should be accepted.

One point to mention is that Malink has no direct textual evidence for his second thesis (p. 126). Malink draws support from other commentators, but I am not sure those commentators endorse precisely the thesis required by Malink’s account. The appeal to Kit Fine (1994) seems misplaced: according to Fine, every necessity has a source in some essence or other, but this does not imply that the subject of predication has an essence in every true predicative necessity-sentence. (Fine himself warns against confusing subject and source of necessity.[3])

Malink’s semantics yield some counter-intuitive results. In many cases, he shows that the result is entailed by Aristotle’s system of validity- and invalidity-claims; no interpretation can validate those claims without yielding the strange result. Some other cases are special to Malink’s semantics. It is important to be aware of these oddities when we weigh the costs and benefits of Malink’s interpretation. Here are a few examples.

In Malink’s main-text interpretation, if ‘A applies to some B’ is true and B is a substance term, then ‘A necessarily applies to some B’ will be true.[4] For example, since ’some humans are smoking’ is true and ‘human’ is a substance term, ‘necessarily, some humans are smoking’ is true.

For some pairs of terms, both ‘A necessarily applies to all B’ and ‘A possibly belongs to no B’ (along with ‘A possibly does not belong to some B’ are true. Malink shows that this is required by Aristotle’s claims of validity and invalidity (pp. 201 ff.). In Malink’s semantics, such pairs of terms are especially easy to come by: one may take any two nonsubstance essence terms which stand to each other as genus to species.[5] For example, both ‘color necessarily belongs to all whiteness’ and ‘color possibly belongs to no whiteness’ are true.

On Malink’s interpretation, there is a systematic ambiguity in Aristotle’s use of the sentence type ‘A necessarily applies to all B’. Within descriptions of syllogistic-style arguments it means one thing (and, as noted above, it is compatible with ‘A possibly does not belong to some B’). But it has a different meaning in other contexts, when Aristotle gives counterexamples to show that a given pair of premises does not yield any syllogistic-style conclusion. In these other contexts, ‘A necessarily applies to all B’ expresses the contradictory of ‘A possibly does not belong to some B’ (p. 213).

Malink takes an extremist approach to his subject matter. Within the modal syllogistic, he accommodates Aristotle’s claims of validity and invalidity of syllogistic-style arguments while making other parts of the text (e.g., Aristotle’s examples and proofs) house all the errors. His treatment of the Topics is also one-sided in certain ways. For example, he tends to focus on the asserted content of passages while downplaying their pragmatic implicature. (An example of this is Malink’s use of Top. 4. 2, 122a10-17 in support of his theses S6 and S7. This passage of the Topics carries, I think, an implicature that goes against Malink’s thesis S3. This is worrying because Malink’s explanation of Barbara NXN relies on the conjunction of S3 and S7.)

Some readers will embrace Malink’s interpretation as it stands — it may, after all, be the best one currently going. Even moderates should see that in many ways the book’s extremism enhances its value, rather than detracting from it. A fully balanced treatment of Aristotle’s modal syllogistic would have to weigh and compromise among so many conflicting factors that it would quickly become bewildering. Malink chooses a method and follows it with tremendous precision, ingenuity, and explicitness. By doing this, he provides an extraordinarily useful point of orientation for any interpreter. Just as the location of a circle's circumference allows us to find its center, Malink's extreme interpretation, well developed and expounded as it is, can help us construct the best possible moderate one.

5.2 THE STRUCTURE OF CATEGORICAL SYLLOGISM

For the time being, let us assume that syllogism means valid categorical syllogism unless otherwise qualified. Syllogism consists of two premises and a conclusion. Thus, we have three prepositions and only three terms. An argument is not syllogistic at all unless it conforms to this structure. Since the number of propositions and terms is three, it is quite obvious that every term occurs twice. Consider an example for a syllogistic argument. 1st premise: All humans are stupid. 2nd premise: All sages are human. Conclusion: Therefore all sages are stupid. A term, which is common to the premises (human), is called middle (M). Predicate of the conclusion (stupid) is called major (P) and subject of the conclusion (sages) is called minor (S). While major has maximum extension, minor has minimum extension. The middle term is so called because its extension varies between the limits set by minor and major. The premise in which major occurs is called major premise and the premise in which minor occurs is called minor premise. Though in this argument the first premise is major and the second is minor there is no rule which stipulates that this must be the order. Not only can minor premise be written first, but also the conclusion can as well be the first statement. The only restriction is that if an argument starts with premises, always 'therefore' or its synonym must precede the conclusion and if the conclusion is the starting point, then 'because' or its synonym must be immediately follow the conclusion. Aristotle argued that our inference proceeds from minor term to major term through middle term. Therefore in the absence of middle term, it is impossible to proceed from minor to major. Aristotle is also a pioneer who discovered predicate logic. He restricted syllogism to subject-predicate logic and, naturally he did not give credence to other forms of proposition like relational prepositions. Most of what Aristotle said on syllogism holds good only when we consider predicate logic.

5.3 AXIOMS OF SYLLOGISM

There are two types of axioms: axioms of quantity and axioms of quality. Rules under these axioms are merely stated because there is no proof to these rules.

A. Axioms of Quantity:

A1: The middle must be distributed at least once in the premise.

A2: A term, which is undistributed in the premise, must remain undistributed in the conclusion. A term, which is distributed in the conclusion, should compulsorily be distributed in the premise.

B. Axioms of quality:

B1: Two negative premises do not yield any conclusion.

B2: Affirmative premises yield only affirmative conclusion.

B3: Negative premise (there can be only one negative premise) yields only negative conclusion.

Three corollaries follow from these rules. They are as follows: -

1. The number of terms distributed in the conclusion must be one less than the number of terms distributed in the premises. It is very easy to explain this corollary. The number of terms in the conclusion itself is one less than the number of terms in the premises and M which is compulsorily distributed in the premises is not a part of the conclusion.

2. Two particular premises do not yield any conclusion. Only one particular premise is permissible.

3. Particular premise yield only particular conclusion. [The reader is advised to prove these corollaries with the help of Axioms of quality and quantity.]

5.4 FIGURES AND MOODS

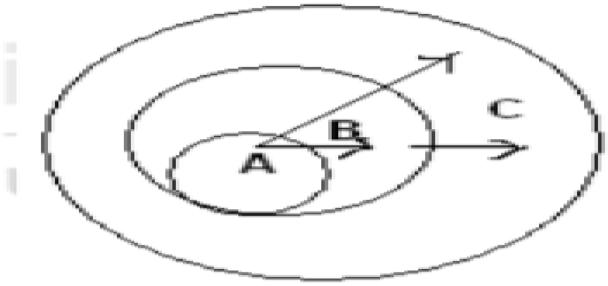
In the conclusion, S and P have fixed positions but this is not the case with M. There are four ways in which M can occupy two places. These four ways are called four figures, i.e., the position of M determines the figure of argument. These figures are as follows: -

	I	II	III	IV
Major Premise:	M-P	P-M	M-P	P-M
Minor Premise:	S-M	S-M	M-S	M-S
Conclusion:	S-P	S-P	S-P	S-P

From this scheme it is clear that neither P nor S determines the figure of syllogism. History has recorded that Aristotle accepted only first three figures. The origin of the fourth figure is disputed. While Quine said that Theophrastus, a student of Aristotle, invented the fourth figure, Stebbing said that it was Gallen who invented the fourth figure. This dispute is not very significant. But what Aristotle says on the first figure is significant. Aristotle regarded the first figure as most ‘scientific’. It is likely that by ‘scientific’ he meant ‘satisfactory’. One of the reasons, which Aristotle has adduced, is that both mathematics and physical sciences establish laws in the form of the first figure. Second reason is that reasoned conclusion or reasoned fact is generally found in the first figure. Aristotle believed that only universal affirmative conclusion can provide complete knowledge and universal affirmative conclusion is possible only in the first figure. Aristotle quotes the fundamental principle of syllogism. ‘One kind of syllogism serves to prove that A inheres in C by showing that A inheres in B and B in C’. This principle can be expressed in this form:

Minor: A inheres in B
Major: B inheres in C
Conclusion: A inheres in C

Evidently, this argument satisfies transitive relation. This is made clear with the help of this diagram:



Let us mention four examples, which correspond to four figures.

FIGURE I

	M P	
Major Premise:	All Artists are Poets.	AAP
	S M	
Minor Premise:	All Musicians are Artists.	MAA
Conclusion:	∴All Musicians are Poets.	MAP
	S P	

FIGURE II

	P M	
Major Premise:	All saints are pious.	SAP
	S M	
Minor Premise:	No criminals are pious.	CEP
Conclusion:	∴ No criminals are saints.	CES
	S P	

FIGURE III

	M P	
Major Premise:	All great works are worthy of study.	GAW
	M S	
Minor Premise:	All great works are epics.	GAE
Conclusion:	∴ Some epics are worthy of study.	EIW

Conclusion:	∴ Some sinners are not soldiers.	SOS
	S P	

We have to consider figures in conjunction with moods. Mood is determined by quality and quantity propositions, which constitute syllogism. Since there are four kinds of categorical proposition and there are three places where they can be arranged in any manner, there are sixtyfour different combinations in any given figure. Since there are four figures, in all, two hundred and fifty six ways of arranging categorical propositions are possible. These are exactly what we mean by moods.

However, out of two hundred and fifty-six, two hundred and forty-five moods can be shown to be invalid by applying the rules and corollaries. So we have only eleven moods. There is no figure in which all eleven moods are valid. In any given figure only six moods are valid. They are as follows:

I.	AAA,	AAI	AII	EAE	EAO	EIO
II.	AEE	AEO	EAE	EAO	EIO	AOO
III.	AAI	AII	IAI	EAO	EIO	OAO
IV.	AAI	IAI	AEE	AEO	EAO	EIO

In all these cases, first letter stands for major premise, second for minor and third for conclusion. Moods are boxed in two ways. Moods within thick boxes are called strengthened moods, and moods within thin boxes are called weakened moods. It is important to know the difference between these two. When two universal premises can yield only particular conclusion, then such moods are called strengthened moods. On the other hand, if we deduce particular conclusion from two universal premises, when it is logically possible to deduce a universal conclusion, then such moods are called weakened moods. When we recall that from universal premises alone particular conclusion cannot be drawn, both strengthened and weakened moods become invalid. Thus, the number of valid moods reduces to fifteen.

In this scheme, we notice that EIO is valid in all the figures. Though EIO is valid in all figures, it is one mood in one figure and some other in another figure. Likewise, AEE is valid in the second and the fourth figures. But it is one mood in the second figure and different mood in the fourth figure. In the thirteenth century, one logician by name Pope John XXI, invented a technique to reduce arguments from other figures to the first figure. This technique is known as mnemonic verses. Accordingly, each mood, excluding weakened moods, was given a special name:

I.	Fig:	AAA	BARBARA	III.	Fig:	AAI	DARAPTI
		EAE	CELARENT			IAI	DISAMIS
		AII	DARII			AII	DATISI
		EIO	FERIO			EAO	FELAPTON
						OAO	BOCARDO
						EIO	FERISON
II.	Fig:	EAE	CESARE	IV.	Fig:	AAI	BRAMANTIP
		AEE	CAMESTRES			AEE	CAMENES
		EIO	FESTINO			IAI	DIMARIS
		AOO	BAROCO			EAO	FESAPO
						EIO	FRESISON

Syllogism can be tested using rules and corollaries. These are also known as general rules. There is one more method of testing syllogism. Every figure is determined by special rules. These are called special rules because they apply only to particular figure. These special rules also depend directly upon the axioms of quantity and quality. Therefore special rules can be proved. While doing so we shall follow the method of reductio ad absurdum because, it is a simple method.

I. Special rules of the first figure: M – P

S – M

S – P

1. Minor must be affirmative:

Proof:

1. Let minor be negative.
2. Conclusion must be negative. (From B3 and 1)
3. Conclusion distributes P. (From 2)
4. Major should distribute P. (From A2 and 3)
5. Major must be negative. (From A2 and 4)
6. Negative minor implies negative major.
7. Two premises cannot be negative (B1)
8. Minor must be affirmative. q.e.d.

2. Major must be universal:

Proof:

1. Let Major be particular.

2. Major undistributes M. (From 1)

3. Minor should distribute M. (From A1 and 1)

4. Minor should be affirmative. (First special rule)

5. Minor has to undistributed M.

6. Major should distribute M. (From A1)

7. Major must be universal. q.e.d. Using these two special rules, valid moods can be distinguished from invalid moods.

II. Special rules of the Second figure: P – M

S – M

S – P

1. Only one premise must be negative:

Proof:

1. Let both premises be affirmative.

2. M is undistributed in affirmative statements.

3. (1) and (2) together contradict A1.

4. One premise must be negative. q.e.d.

2. Major should be universal:

Proof:

1. Let Major be particular.

2. Major undistributes P. (from 1)

3. Conclusion must be universal. (From B3 and first special rule).

4. Conclusion distributes P.

5. (2) and (4) together contradict A2.

6. Major should distribute P.

7. Major must be universal.

III. Special rules of the Third figure: M – P

M – S

S – P

1. Minor must be affirmative.

2. Conclusion must be particular.

(The reader is advised to try to prove these two rules).

IV. Special rules of the Fourth figure: P – M

M – S

S – P

1. If Major is affirmative, then minor must be universal.

Proof:

1. Let minor be particular when major is affirmative.
2. Major undistributes M.
3. Minor also undistributes M. (From 1)
4. (2) and (3) together contradict A1.
5. Minor should distribute M.
6. Minor must be universal.

2. If any premise is negative, major must be universal.

Proof:

1. Let major be particular, when one premise is negative.
2. Negative premise yields negative conclusion. (B3)
3. Negative conclusion distributes P.
4. Major should distribute P. (From 3 and A2)
5. Major must be universal.
6. (1) and (5) contradict one another.

7 Major must be universal. q.e.d.

3. If minor is affirmative, then, conclusion must be particular.

Proof:

1. Let conclusion be universal with affirmative minor.
2. Universal conclusion distributes S.
3. Minor should distribute S. (From A2 and 2)
4. Affirmative minor undistributed S.
5. (3) and (4) contradict one another.
6. Conclusion should undistribute S.
7. Conclusion must be particular.

5.5 FALLACIES

There are three important fallacies associated with categorical syllogism. They are fallacies of undistributed middle, illicit major and illicit minor. One example for each fallacy with explanation will suffice.

P M

Major Premise: All inscriptions are contents of historical study. IAC

S M

Minor Premise: All ancient coins are contents of historical study. AAC

Conclusion: All ancient coins are inscriptions. AAI

Ans: This argument is in the second figure. According to one special rule of the second figure, only one premise must be negative. Since this rule is violated M is undistributed in both the premises. The argument commits the fallacy of undistributed middle.

While mentioning the rule violated we can also say that according to one axiom of quantity, M should be distributed at least once. When this rule is violated this fallacy is committed.

M P

Major Premise: All sailors are strong. SAS

M S

Minor Premise: All sailors are men. SAM

S P

Conclusion: All men are Strong. MAS

Ans: This argument is in the third figure. According to one special rule of the third figure, the conclusion must be particular. Since this rule is violated, the argument commits the fallacy of illicit minor. [The reader is advised to identify the second type of explanation.]

P M

Major Premise: Some rich people are merchants. RIM

M S

Minor Premise: No merchants are educated. MEE

Conclusion: Some educated persons are not rich. EOR

Ans: This argument is in the fourth figure. According to one special rule of the fourth figure, when a premise is negative major must be universal. This rule is violated by the argument and it commits the fallacy of illicit major. [The reader is advised to identify the second type of explanation.]

In any deductive argument certain elements are constant. In syllogism, for example, quality and quantity and position of terms determine the structure of the argument. Keeping the structure constant if any term is replaced by any other term, the end result remains the same. Therefore the student can construct as many examples as he or she wants. The method of identifying the fallacy remains the same, if the structure remains the same.

5.6 REDUCTION OF ARGUMENTS

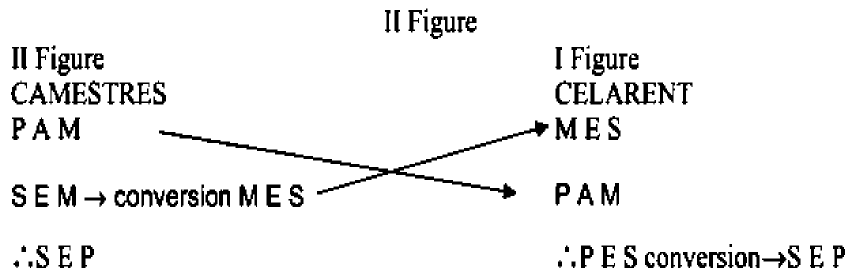
Reducing arguments from other figures to the first figure is one of the techniques developed by Aristotle to test the validity of arguments. It is because Aristotle held that the first figure is the perfect one; all others are imperfect. After reduction, if the argument is valid in the first figure, then it means that the original argument in any other figure is valid. This technique is quite mechanical. So, we are only required to know what exactly is the method involved. We will learn this only by practice.

II Figure

II Figure CESARE		I Figure CELARENT
P E M	→ Conversion→	M E P
S A M		S A M
S E P		S E P

No politicians are poets.	→ Conversion →	No poets are politicians.
All girls are poets.		All girls are poets.
∴No girls are politicians.		∴No girls are politicians.

In CESARE ‘S’ after ‘E’ indicates simple conversion. It shows that ‘E’ (major premise) must undergo simple conversion.



‘S’ and ‘T’ after ‘E’ shows that ‘E’ (minor premise) should undergo simple conversion and both premises be transposed. ‘S’ after second ‘E’ shows that this ‘E’ (conclusion) should undergo simple conversion. [The student is advised to construct argument for this and subsequent reductions.]

P E M → Conversion→ M E P
M A S → Conversion→ S I M
SOP SOP

As usual ‘S’ stands for simple conversion of ‘E’ (Major Premise) and ‘P’ stands for conversion per accedens of ‘A’ (Minor premise). This process is similar to first and third moods of III figure.

FRESISON FERIO
P E M → Conversion→ M E P
M I S → Conversion→ S I M
S O P S O P

A close observation of the above reductions reveals that they are to be performed according to certain parameters. The moods in the first figure are Barbara, Celarent, Darii and Ferio. Their initial consonants are arbitrarily found. For other figures, the initial consonants indicate to which of the first, the figure is to be reduced. Accordingly, Fesapo in the 4th figure is to be reduced to Ferio. Other consonants occurring in second, third and fourth figures' mnemonics indicate the operation that must be performed on the proposition indicated by the preceding vowel in order to reduce the syllogism to a first-figure syllogism. Certain 'keys' are the following. 's' indicates simple conversion; 'p' indicates conversion per accidens (by limitation); 'm' indicates the interchanging of the premises; 'k' indicates obversion; 'c' refers to the process that the syllogism is to be reduced indirectly. Meaningless letters in mnemonic terms are 'r', 't', 'l', 'n', and noninitial 'b' and 'd'. From reduction technique one point becomes clear. Originally, there were twenty-four valid moods. Later weakened and strengthened moods were eliminated on the ground that particular proposition (existential quantifier) cannot be deduced from universal propositions (universal quantifier) alone, and the number was reduced to fifteen. Now after reduction to first figure the number came down to four. Strawson argues that reduction technique is superior to axiomatic technique to which he referred in the beginning of his work 'Introduction to Logical Theory'. He regards the moods as inference-patterns. He argues that the path of reduction should be an inverted pyramid. At one particular point of time Strawson maintains that in addition to equivalence relation, we require opposition relation also to effect reduction.

5.7 ANTILOGISM OR INCONSISTENT TRIAD

This technique was developed by one lady by name, Christine Ladd-Franklin (1847-1930). This technique applies only to fifteen moods. The method is very simple. Consider Venn's results for all propositions. Replace the conclusion by its contradiction. This arrangement constitutes antilogism. If the corresponding argument should be valid, then antilogism should conform to certain structure. It must possess two equations and one inequation. A term must be common to equations. It should be positive in one equation and negative in another. Remaining two terms appear in inequation. Consider one example for a valid argument.

There are five techniques to test the validity of arguments. Conditions of validity differ from traditional analysis to modern analysis. There are three important fallacies in this category.

The Inconsistent Triad is an argument against the concept of an all-powerful and all-loving God whilst suffering persists. The existence of suffering alongside an all-loving (omnibenevolent) and all-powerful (omnipotent) God are argued to be contradictory. The God the argument is posed against is typically the Judeo-Christian God. The argument is as follows; if God is all-loving and all-powerful he should be able to prevent any suffering. Suffering, however, exists. From this, it is concluded that God either cannot be omnipotent or cannot be omnibenevolent. It is called the Inconsistent Triad because it is comprised of three states of existence (making three corners of a triangle) that supposedly cannot co-exist.

Challenges

The existence of evil and suffering is a significant problem for religious people who have tried to understand and explain their prescence.

If someone is not religious, then evil is just part of our world and has to be accepted - there is nothing we can do about it. However, for religious people there are significant questions:

- Religions such as Christianity claim that God made everything. Does that mean He also made evil?
- Religion teaches that God is good, so why does God allow evil to exist?
- If God is powerful enough to create the world, why does He not stop evil and suffering? Is He not powerful enough?
- If God is all powerful, does that mean He does not love us enough to stop evil and suffering?
- If evil exists, does God really exist?

Epicurus

The Greek philosopher **Epicurus (342-271 BCE)** claimed that the existence of God proved there is no God.

He claimed that if God cannot stop evil then he is not all-powerful (omnipotent).

He then argued that if God can prevent evil but does not, then God is not good.

He linked these two points together, claiming that if God is all-powerful and good, then evil would not exist.

Finally, human experience is that evil does exist. Therefore Epicurus concluded that God must not exist.

The inconsistent triad

The problem of evil can be regarded as an **'inconsistent triad'** – in other words, **three** ideas but only **two** of them can be true.

As there is clear evidence and experience of evil, either God is not all-powerful (ie He cannot stop evil) or God is not loving and good (ie He does not love us or care enough to stop evil).

The inconsistent triad

Some people believe that if evil exists and God is all-powerful, then He cannot be all loving.

The problem of evil is often given in the form of an inconsistent triad. For example, J. L. Mackie gave the following three propositions:

God is omnipotent

God is omnibenevolent

Evil exists

Mackie argued that these propositions were inconsistent, and thus, that at least one of these propositions must be false. Either:

1. God is omnipotent and omnibenevolent, and evil does not exist.
2. God is omnipotent, but not omnibenevolent; thus, evil exists by God's will.
3. God is omnibenevolent, but not omnipotent; thus, evil exists, but it is not within God's power to stop it (at least not instantaneously).

Now number one doesn't seem to be a very good option. Evil or suffering is something that everyone can get on board with accepting as true.

Number two and number three must be held to be false by serious theists.

Therefore, the theist will conclude that all three are false.

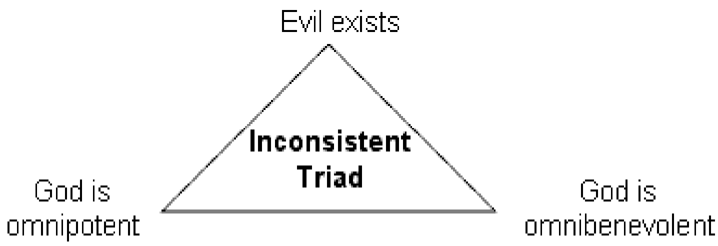
All the theist needs to do is say:

A. God is omnipotent meaning he can do only things that are logically possible.

-If one thinks God can do logically impossible things, then the logical problem of evil vanishes since the whole problem is predicated on the idea that there is a logical absurdity or contradiction with God and evil both existing.

B. God is omnibenevolent, which means he will eliminate evil, unless he has a morally sufficient reason for allowing evil.

-If it’s even logically possible that God has a morally sufficient reason for allowing evil, then there is no problem with asserting that God and evil exists.



5.8 VENN DIAGRAM TECHNIQUE

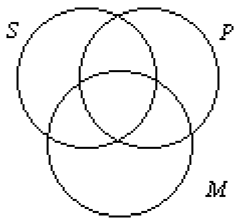
I. One good method to test quickly syllogisms is the Venn Diagram technique. This class assumes you are already familiar with diagramming categorical propositions. You might wish to review these now: Venn Diagrams.

A. A syllogism is a two premiss argument having three terms, each of which is used twice in the argument.

B. Each term (major, minor, and middle terms) can be represented by a circle.

C. Since a syllogism is **valid** if and only if the premisses entail the conclusion, diagramming the premisses will reveal the logical geography of the conclusion in a valid syllogism. If the syllogism is **invalid**, then diagramming the premisses is insufficient to show the conclusion must follow.

D. Since we have three classes, we expect to have three overlapping circles.



1. The area in the denoted circle represents where members of the class would be, and the area outside the circle represents all other individuals (the complementary class). The various area of the diagram are noted above.

2. Shading represents the knowledge that no individual exists in that area. Empty space represents the fact that no information is known about that area.

3. An "X" represents "at least one (individual)" and so corresponds with the word "some."

II. Some typical examples of syllogisms are shown here by their mood and figure.

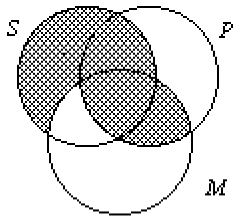
A. **EAE-1**

1. The syllogism has an **E** statement for its major premiss, an **A** statement for its minor premiss, and an **E** statement for its conclusion. By convention the conclusion is labeled with **S** (the minor term) being the subject and **P** (the major term) being the predicate. The position of the middle term is the "left-hand wing."

2. The form written out is
No **M** is **P**.
All **S** is **M**
No **S** is **P**.

3. Note, in the diagram below, how the area in common between S and P has been completely shaded out indicating that "No **S** is **P**."

The conclusion has been reached from diagramming only the two premisses. All syllogisms of the form **EAE**-1 are valid.

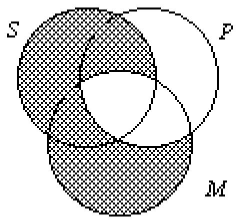


B. **AAA**-1

1. This syllogism is composed entirely of "**A**" statements with the **M**-terms arranged in the "left-hand wing" as well.

2. Its form is written out as
All **M** is **P**.
All **S** is **M**.
All **S** is **P**.

3. Note, in the diagram below, how the only unshaded area of **S** is in all three classes. The important thing to notice is that this area of **S** is entirely within the **P** class. Hence, the **AAA**-1 syllogism is always valid. In ordinary language the **AAA**-1 and the **EAE**-1 syllogisms are by far the most frequently used.



C. **AII**-3

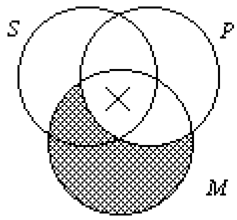
1. The **AII**-3 syllogism has the **M**-terms arranged in the subject position--the right side of the brick.

2. This syllogism sets up as
All **M** is **P**.
Some **M** is **S**.
Some **S** is **P**.

3. When diagramming the syllogism, notice how you are "forced" to

put the "**X**" from the minor premiss in the area of the diagram shared by all three classes. The "**X**" cannot go on the **P**-line because the shading indicates this part of the SM area is empty. This "logical" forcing enables you to read-off the conclusion, "Some **S** is **P**."

4. This syllogism is a good example why the universal premiss should be diagrammed before diagramming a particular premiss. If we were to diagram the particular premiss first, the "**X**" would go on the line. Then, we would have to move it when we diagram the universal premiss because the universal premiss empties an area where the "**X**" could have been.



D. **AII**-2

1. The **AII**-2 has the **M** terms in the predicate of both premisses.

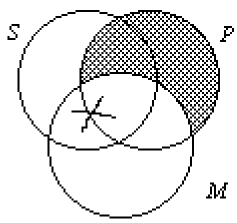
2. The syllogism is written out as

All **P** is **M**.

Some **S** is **M**.

Some **S** is **P**.

3. The diagram below shows that the "**X**" could be in the **SMP** area or in the **SPM** area. Since we do not know exactly which area it is in, we put the "**X**" on the line, as shown. When an "**X**" is on a line, we do not know with certainty exactly where it is. So, when we go to read the conclusion, we do not know where it is. Since the conclusion cannot be read with certainty, the **AII**-2 syllogism is invalid.



Notes

E. The final syllogism described here, the **EAO**-4 raises some interesting problems.

1. Notice that in this syllogism there are universal premisses with a particular conclusion.

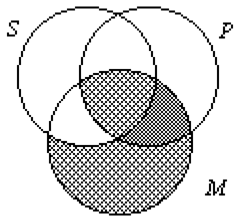
2. Its form is written out as

No **P** is **M**.

All **M** is **S**.

Some **S** is not **P**.

3. And its diagram is rather easily drawn as



4. When we try to read the conclusion, we see that there is no "**X**" in the **SMP** class. We must conclude that the syllogism is invalid because we cannot read-off "Some **S** is not **P**."

5. However, if we know that **M** exists, all the members of **M** have to be in the **SMP** class. These **M**'s are **S**'s as well. Hence, we know that some **S**'s are not **P**'s! In other words, the **EOA**-4 syllogism is valid if we know ahead of time the additional premiss "**M** exists."

6. Most contemporary logicians have concluded that we should not assume any class exists unless we have evidence.

a. We want to talk about theoretical entities without assuming their existence.

b. For example, in science and mathematics, our logic will apply when talking about circles, points, frictionless planes, and freely falling bodies even though these entities do not physically exist.

c. This diagram illustrates the contemporary topic called **the problem of existential import**. When can we reasonably conclude something exists? How does this conclusion affect our theory of

			logical validity?

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Discuss about Syllogism?

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

5.9 LET US SUM UP

There is a long tradition of scholarship that treats Aristotle's logic as if there are no restrictions on what we choose as our modal syllogistic terms. I do not think that is right; I think Aristotle himself often relies on what are ultimately semantic considerations when he accepts and rejects modal syllogisms. My aim in the present paper is to show that the right semantic restrictions on the principles of modal conversion are powerful enough to do all the work. So, instead of taking the Kneales' point as an objection to Aristotle, I want to take their point as a starting place and suggest that 'peculiar predicates' are exactly what Aristotle has in mind. If substituting modal for nonmodal terms in assertoric syllogisms can get us exactly those syllogisms Aristotle says are valid and none of the ones he says are not, then there is a sense in which the Kneales are right and the modal syllogistic is trivial. Of course, if it is trivial in this sense, then we will not need anything like (1) and (2) in order to capture Aristotle's meaning. Instead, any modal rules should be only modal versions (that is, substitution instances) of ordinary nonmodal rules, so there is no separate logic of modality for Aristotle. There is only one logic.

5.10 KEY WORDS

Paradox: A paradox is a statement or group of statements that leads to a contradiction or a situation which defies intuition or common experience.

5.11 QUESTIONS FOR REVIEW

1. Discuss about The Structure of Categorical Syllogism.
2. What is Axioms of Syllogism?
3. Discuss about the Figures and Moods.
4. What is Fallacies?
5. Discuss the Reduction of Arguments.
6. What is Antilogism or Inconsistent Triad?
7. What is Venn Diagram Technique?

5.12 SUGGESTED READINGS AND REFERENCES

- Alexander, P. An Introduction to Logic & The Criticism of arguments. London: Unwin University, 1969.
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5.13 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. The rule which is common to conversion and syllogism is: ‘term which is undistributed in the premise must remain undistributed in the conclusion’.

UNIT 6: STOIC TREATMENT OF MODALITY

STRUCTURE

6.0 OBJECTIVES

After this unit, we can able to know:

- To know the Sources of our information on the Stoics
- To discuss the Philosophy and life
- To know about the Physical Theory
- To discuss the Logic
- To know the Ethics
- To describe the Influence

6.1 INTRODUCTION

Stoicism was one of the new philosophical movements of the Hellenistic period. The name derives from the porch (stoa poikilê) in the Agora at Athens decorated with mural paintings, where the members of the school congregated, and their lectures were held. Unlike 'epicurean,' the sense of the English adjective 'stoical' is not utterly misleading with regard to its philosophical origins. The Stoics did, in fact, hold that emotions like fear or envy (or impassioned sexual attachments, or passionate love of anything whatsoever) either were, or arose from, false judgements and that the sage – a person who had attained moral and intellectual perfection – would not undergo them. The later Stoics of Roman Imperial times, Seneca and Epictetus, emphasise the doctrines (already central to the early Stoics' teachings) that the sage is utterly immune to misfortune and that virtue is sufficient for happiness. Our phrase 'stoic calm' perhaps encapsulates the general drift of these claims. It does not, however, hint at the even more radical ethical views which the Stoics defended, e.g. that only the sage is free while all others are slaves, or that all those who are morally vicious are equally so. (For other examples, see Cicero's brief essay 'Paradoxa Stoicorum'.) Though it seems clear that some Stoics took a kind of perverse joy in advocating views which seem so at odds with common sense, they did not do so simply to shock. Stoic ethics achieves a certain plausibility within the context of their physical theory and psychology, and within the framework of Greek ethical theory as that was handed down to them from Plato and Aristotle. It seems that they were well aware of the mutually interdependent nature of their philosophical views, likening philosophy itself to a living animal in which logic is bones and sinews; ethics and physics, the flesh and the soul respectively (another version reverses this assignment, making ethics the soul). Their views in logic and physics are no less distinctive and interesting than those in ethics itself.

6.2 SOURCES OF OUR INFORMATION ON THE STOICS

Since the Stoics stress the systematic nature of their philosophy, the ideal way to evaluate the Stoics' distinctive ethical views would be to study them within the context of a full exposition of their philosophy. Here, however, we meet with the problem about the sources of our knowledge about Stoicism. We do not possess a single complete work by any of the first three heads of the Stoic school: the 'founder,' Zeno of Citium in Cyprus (344–262 BCE), Cleanthes (d. 232 BCE) or Chrysippus (d. ca. 206 BCE). Chrysippus was particularly prolific, composing over 165 works, but we have only fragments of his works. The only complete works by Stoic philosophers that we possess are those by writers of Imperial times, Seneca (4 BCE–65 CE), Epictetus (c. 55–135) and the Emperor Marcus Aurelius (121–180) and these works are principally focused on ethics. They tend to be long on moral exhortation but give only clues to the theoretical bases of the moral system. For detailed information about the Old Stoa (i.e. the first three heads of the school and their pupils and associates) we have to depend on either doxographies, like pseudo-Plutarch Philosophers' Opinions on Nature, Diogenes Laertius' Lives of Eminent Philosophers (3rd c. CE), and Stobaeus' Excerpts (5th c. CE) – and their sources Aetius (ca. 1st c. CE) and Arius Didymus (1st c. BCE–CE) – or other philosophers (or Christian apologists) who discuss the Stoics for their own purposes. Nearly all of the latter group are hostile witnesses. Among them are the Aristotelian commentator Alexander of Aphrodisias (late 2nd c. CE) who criticises the Stoics in On Mixture and On Fate, among other works; the Platonist Plutarch of Chaeronea (1st-2nd c. CE) who authored works such as On Stoic Self-Contradictions and Against the Stoics on Common Conceptions; the medical writer Galen (2nd c. CE), whose outlook is roughly Platonist; the Pyrrhonian skeptic, Sextus Empiricus (2nd c. CE); Plotinus (3rd c. CE); the Christian bishops Eusebius (3rd–4th c. CE) and Nemesius (ca. 400 CE); and the sixth-century neoplatonist commentator on Aristotle, Simplicius. Another important source is Cicero (1st c. BCE). Though his own philosophical position derives from that of his teacher Philo of Larissa and the New Academy, he is not without sympathy for what he sees as the high moral tone of Stoicism. In works like his Academic Books, On the Nature of the Gods, and On Ends he provides summaries in Latin, with critical discussion, of the views of the major Hellenistic schools of thought.

From these sources, scholars have attempted to piece together a picture of the content, and in some cases, the development of Stoic doctrine. In some areas, there is a fair bit of consensus about what the Stoics thought and we can even attach names to some particular innovations. However, in other areas the proper interpretation of our meagre evidence is hotly contested. Until recently, non-specialists have been largely excluded from the debate because many important sources were not translated into modern languages. Fragments of Stoic works and testimonia in their original Greek and Latin were collected into a three-volume set in 1903–5 by H. von Arnim, Stoicorum Veterum Fragmenta. In writings on the 'old' Stoics, fragments and testimonia are often referred to by von Arnim's volume numbers and text numeration; e.g. SVF I.345=Diogenes Laertius, Lives 4.40. In 1987, A. A. Long and David Sedley brought out The Hellenistic Philosophers (LS) which contains in its first volume English translations and commentary of many important texts on Stoics, Epicureans and Skeptics. In 1988 Long and Sedley was followed by a collection of primary texts edited by B. Inwood and L. P. Gerson entitled Hellenistic Philosophy. Unless otherwise specifically noted, I refer in what follows to texts by or about Stoics using the author's name followed by Long and Sedley's notation for the text. For example, 'Aetius, 26A' refers to section 26 of Aetius's work, text A (unless otherwise noted, I use their translation, sometimes slightly altered). The Inwood and Gerson collection translates many of the same texts, but unlike LS does not chop them up into smaller bits classified by topic. Each approach has its merits, but the LS collection better serves the needs of an encyclopedia entry. For French translation of Chrysippus, see Dufour (2004). For German translation of the early Stoa, see Nickel (2009).

6.3 PHILOSOPHY AND LIFE

When considering the doctrines of the Stoics, it is important to remember that they think of philosophy not as an interesting pastime or even a particular body of knowledge, but as a way of life. They define philosophy as a kind of practice or exercise (askêsis) in the expertise concerning what is beneficial (Aetius, 26A). Once we come to know what we and the world around us are really like, and especially the nature of value, we will be utterly transformed. This therapeutic aspect is common to their main competitors, the Epicureans, and perhaps helps to explain why both were eventually eclipsed by Christianity. The Meditations of Marcus Aurelius provide a fascinating picture of a would-be Stoic sage at work on himself. The book, also called To Himself, is the emperor's diary. In it, he not only reminds himself of the content of important Stoic teaching but also reproaches himself when he realises that he has failed to incorporate this teaching into his life in some particular instance. Today many people still turn to Stoicism as a form of psychological discipline. Stoicism has never been ‘purely academic’ and modern adaptations of Stoic thought seek to carry on this tradition of self-transformation. One of the most influential modern interpretations of means through which the Stoic philosophizing accomplished such a transformation introduces the notion of spiritual exercises. Hadot (1998) provides a reading of Marcus Aurelius’ Meditations as a set of such exercises. For a more general treatment covering Stoic philosophy as a whole, see Sellars (2013). For a recent discussion of the entire question of philosophy as a way – or rather as many ways – of life in antiquity, see Cooper 2102.

6.4 PHYSICAL THEORY

An examination of Stoic ontology might profitably begin with a passage from Plato’s Sophist (cf. Brunschwig 1994). There (247d-e), Plato asks for a mark or indication of what is real or what has being. One answer which is mooted is that the capacity to act or be acted upon is the distinctive mark of real existence or ‘that which is.’ The Stoics accept this criterion and add the rider that only bodies can act or be acted upon. Thus, only bodies exist. So there is a sense in which the Stoics are materialists or – perhaps more accurately, given their understanding of matter as the passive principle (see below) – ‘corporealists’. However, they also hold that there are other ways of appearing in the complete inventory of the world than by virtue of existing. Incorporeal things like time, place or sayables (lekta, see below) are ‘subsistent’ (huphestos, Galen 27G) – as are imaginary things like centaurs. The distinction between the subsistent and the existent somewhat complicates the easy assimilation of Stoicism to modern materialism. It’s not wrong to say that all existent things are corporeal according to the Stoics, but one needs to add that existent things don’t exhaust their ontology. All existent things are, in addition, particulars. The Stoics call universals ‘figments of the mind’. Platonic Forms, in particular, are rejected as ‘not somethings’ which lack even the subsistent status of incorporeals like time, place or sayables (Alexander, 30D). The Stoics' positive nominalist alternative is harder to interpret. Some texts suggest that they offered a conceptualist treatment akin to Locke's, treating an apparent predication like "man is a rational, mortal animal" as the disguised conditional, "if something is a man, then it is a rational mortal animal" (Sextus Empiricus, 30I). But there may well have been development within the school from this conceptualist view toward a form of predicate nominalism. See Caston (1999).

In accord with this ontology, the Stoics, like the Epicureans, make God a corporeal entity, though not (as with the Epicureans) one made of everyday matter. But while the Epicureans think the gods are too busy being blessed and happy to be bothered with the governance of the universe (Epicurus, Letter to Menoeceus 123–4), the Stoic God is immanent throughout the whole of creation and directs its development down to the smallest detail. The governing metaphor for Stoic cosmology is biological, in contrast to the fundamentally mechanical conception of the Epicureans. The entire cosmos is a living thing and God stands to the cosmos as an animal's life force stands to the animal's body, enlivening, moving and directing it by its presence throughout. The Stoics insistence that only bodies are capable of causing anything, however, guarantees that this cosmic life force must be conceived of as somehow corporeal.

More specifically, God is identical with one of the two ungenerated and indestructible first principles (archai) of the universe. One principle is matter which they regard as utterly unqualified and inert. It is that which is acted upon. God is identified with an eternal reason (logos, Diog. Laert. 44B) or intelligent designing fire or a breath (pneuma) which structures matter in accordance with Its plan (Aetius, 46A). The designing fire is likened to sperm or seed which contains the first principles or directions of all the things which will subsequently develop (Aristocles in Eusebius, 46G). The biological conception of God as a kind of living heat or seed from which things grow seems to be fully intended. The further identification of God with pneuma or breath may have its origins in medical theories of the Hellenistic period. See Baltzly (2003). On the entire issue of God and its relation to the cosmos in Stoicism, see the essays in Salles (2009).

Just as living things have a life-cycle that is witnessed in parents and then again in their off-spring, so too the universe has a life cycle that is repeated. This life cycle is guided by, or equivalent to, a developmental plan that is identified with God. There is a cycle of endless recurrence, beginning from a state in which all is fire, through the generation of the elements, to the creation of the world we are familiar with, and eventually back to the state of pure designing fire called 'the conflagration' (Nemesius, 52C). This idea of world-cycles punctuated by conflagrations raised a number of questions. Will there be another you reading this encyclopedia entry in the next world cycle? Or merely someone exactly similar to you? Different sources attribute different answers to the Stoics on these questions. (For sameness of person, see Alexander (52F). For someone indistinguishable, but not not identical, see Origen (52G).) The doctrine of eternal recurrence also raises interesting questions about the Stoic view of time. Did they suppose that the moment in the next world cycle at which you (or someone indistinguishable from you) reads this entry is a moment in the future (so time is linear) or the very same moment (with some notion of circular time)? The Stoic definition of time as the 'dimension (diastêma) of motion' or 'of the world's motion' (Simplicius, 51A) hardly seems to settle the question. For a clear exchange on the issue, see Long (1985) and Hudson (1990).

The first things to develop from the conflagration are the elements. Of the four elements, the Stoics identify two as active (fire and air) and two as passive (water and earth). The active elements, or at least the principles of hot and cold, combine to form breath or pneuma. Pneuma, in turn, is the 'sustaining cause' (causa continens, synektikon aition) of all existing bodies and guides the growth and development of animate bodies. What is a sustaining cause? The Stoics think that the universe is a plenum. Like Aristotle, they reject the existence of empty space or void (except that the universe as a whole is surrounded by it). Thus, one might reasonably ask, 'What marks any one object off from others surrounding it?' or, 'What keeps an object from constantly falling apart as it rubs elbows with other things in the crowd?'. The answer is: pneuma. Pneuma, by its nature, has a simultaneous movement inward and outward which constitutes its inherent 'tensility.' (Perhaps this was suggested by the expansion and contraction associated with heat and cold.) Pneuma passes through all (other) bodies; in its outward motion it gives them the qualities that they have, and in its inward motion makes them unified objects (Nemesius, 47J). In this latter respect, pneuma plays something like one of the roles of substantial form in Aristotle for this too makes the thing of which it is the form 'some this,' i.e. an individual (Metaph. VII, 17). Because pneuma acts, it must be a body and it appears that the Stoics stressed the fact that its blending with the passive elements is 'through and through' (Galen 47H, Alex. Aph. 48C). Perhaps as a result of this, they developed a theory of mixture which allowed for two bodies to be in the same place at the same time. It should be noted, however, that some scholars (e.g. Sorabji, 1988) think that the claim that pneuma is blended through the totality of matter is a conclusion that the Stoics' critics adversely drew about what some of their statements committed them to. Perhaps instead they proposed merely that pneuma is the matter of a body at a different level of description.

Pneuma comes in gradations and endows the bodies which it pervades with different qualities as a result. The pneuma which sustains an inanimate object is (LS) a 'tenor' (hexis, lit. a holding). Pneuma in plants is, in addition, (LS) physique (phusis, lit. 'nature'). In animals, pneuma is soul (psychê) and in rational animals pneuma is, besides, the commanding faculty (hêgemonikon) (Diog. Laert. 47O, Philo 47P) – that is responsible for thinking, planning, deciding. The Stoics assign to 'physique' or 'nature' all the purely physiological life functions of a human animal (such as digestion, breathing, growth etc.) – self-movement from place to place is due to soul.

Their account of the human soul (mind) is strongly monistic. Though they speak of the soul's faculties, these are parts of the commanding faculty associated with the physical sense organs (Aetius, 53H). Unlike the Platonic tri-partite soul, all impulses or desires are direct functions of the rational, commanding faculty. This strongly monistic conception of the human soul has serious implications for Stoic epistemology and ethics. In the first case, our impressions of sense are affections of the commanding faculty. In mature rational animals, these impressions are thoughts, or representations with propositional content. Though a person may have no choice about whether she has a particular rational impression, there is another power of the commanding faculty which the Stoics call ‘assent’ and whether one assents to a rational impression is a matter of volition. To assent to an impression is to take its content as true. To withhold assent is to suspend judgement about whether it is true. Because both impression and assent are part of one and the same commanding faculty, there can be no conflict between separate and distinct rational and nonrational elements within oneself – a fight which reason might lose. Compare this situation with Plato’s description of the conflict between the inferior soul within us which is taken in by sensory illusions and the calculating part which is not (Rep. X, 602e). There is no reason to think that the calculating part can always win the epistemological civil war which Plato imagines to take place within us. But because the impression and assent are both aspects of one and the same commanding faculty according to the Stoics, they think that we can always avoid falling into error if only our reason is sufficiently disciplined. In a similar fashion, impulses or desires are movements of the soul toward something. In a rational creature, these are exercises of the rational faculty which do not arise without assent. Thus, a movement of the soul toward X is not automatically consequent upon the impression that X is desirable. This is what the Stoics’ opponents, the Academic Skeptics, argue against them is possible (Plutarch, 69A.) The Stoics, however, claim that there will be no impulse toward X – much less an action – unless one assents to the impression (Plutarch, 53S). The upshot of this is that all desires are not only (at least potentially) under the control of reason, they are acts of reason. Thus there could be no gap between forming the decisive judgement that one ought to do X and an effective impulse to do X.

Since pneuma is corporeal, there is a sense in which the Stoics have a theory of mind that would be called ‘materialist’ in the modern sense (cf. Annas 2009). The pneuma which is a person’s soul is subject to generation and destruction (Plutarch 53 C, Eusebius 53W). Unlike for the Epicureans, however, it does not follow from this that my soul will be utterly destroyed at the time at which my body dies. Chrysippus alleged that the souls of the wise would not perish until the next conflagration (Diog. Laert. 7.157=SVF 2.811, not in LS). Is this simply a failure of nerve on the part of an otherwise thorough-going materialist? Recall that the distinctive movement of pneuma is its simultaneous inward and outward motion. It is this which makes it tensile and capable of preserving, organising and, in some cases, animating the bodies which it interpenetrates. The Stoics equate virtue with wisdom and both with a kind of firmness or tensile strength within the commanding faculty of the soul (Arius Didymus 41H, Plutarch 61B, Galen 65T). Perhaps the thought was that the souls of the wise had a sufficient tensile strength that they could continue to exist as a distinct body on their own. Later Stoics like Panaetius (2nd c. BCE) and Posidonius (first half 1st c. BCE) may have abandoned this view of Chrysippus'.

Let us conclude this survey of the physical part of Stoic philosophy with the question of causal determinism, though this is an issue that will emerge again in the following section on logic. The clear first impression of Stoic philosophy is that they are determinists about causation, who regard the present as fully determined by past events, but who nonetheless want to preserve scope for moral responsibility by defending a version of compatibilism. That characterisation is not wrong exactly, but it makes the matter sound far simpler than it in fact is since it effaces some important differences between our framework for discussing these matters and that of ancient philosophers. One key difference is that most contemporary thinking about causation treats it as a relation between events. But ancient discussions of causation take place in a context that has no ready vocabulary for events. That doesn't mean they denied the existence of events or failed to notice that things happen. It just means that there is no specific piece of philosophical terminology for contrasting what happens with the things that it happens to or with truths about what happened. When we speak of events, we speak of things that helpfully fill the gap between things and statements. Since they take place at a particular time and involve some objects and not others, events are somewhat thing-like. (Even theories of events that don't treat them as concrete particulars must in some way do justice to this aspect of event talk.) On the other hand, they also have a propositional structure of sorts.

The event of Seneca sitting in a bathtub contemplating a book involves such objects as Seneca, his book and his tub, but it involves them in a way that has a kind of structure. Though it involves the same objects, the event of Seneca sitting on his book and contemplating his bathtub is very different from the first event. Absent a robust concept of causation as a relation among events, Stoic analyses of causation sound very odd to the modern ear.

The Stoics say that every cause is a body which becomes the cause to a body of something incorporeal. For instance the scalpel, a body, becomes the cause to the flesh, a body, of the incorporeal predicate ‘being cut’. (Sextus, 55B; cf. Stobaeus, 55A)

The propositional event-like structure of the effect in the Stoic account of causation is given by the insistence that the cause brings it about that a body has a predicate true of it. But if we think about Sextus’ example, it’s not just the scalpel that is the cause of the flesh being cut, for it did not cause this when it was stored safely in a drawer. So there’s event-like structure (the flesh’s being cut) on the effect side of the causal interaction in the Stoic analysis, but not on the cause side. There we have just the body, the scalpel. The role of the event-like structure of the cause in the Stoic scheme is fulfilled by talking about a whole range of different kinds of causes. The sources on the Stoic taxonomy of causes are complex and conflicting, so we can confine our attention to a few of the more important kinds: preliminary causes, sustaining causes, and proximate causes. In this respect, the Stoic view is not wholly unlike Aristotle’s account which famously included ‘the four causes’. It would be more accurate to say that Aristotle’s four causes are four kinds of explanatory factors. What Aristotle does not say, however, is that the presence of these explanatory factors necessitates that which they explain. Causal processes involve a kind of generality for Aristotle. They bring about these things ‘always or for the most part’ but that is very different from the Stoic insistence that causes necessitate. The working out of the divine plan by God or the world’s pneuma they call ‘fate’ and describe it as a sequence of causes that is ‘inescapable’ (aparabatos Aetius, 55J; cf. Gellius, 55K). In On Fate Cicero sought to explain how Chrysippus attempted to avoid the conclusion that, since our actions come about by prior causes, they are not in our power. Chrysippus' answer turns on the different kinds of causes the Stoics sought to identify and it is worth quoting at length:

But Chrysippus, disapproving of necessity and at the same time wanting nothing to happen without antecedent causes, distinguishes between the kinds of cause, in order to escape necessity while retaining fate. "Of causes", he explains, "some are complete and primary, others auxiliary and proximate. Hence when we say that all things come about through fate by antecedent causes, we do not mean this to be understood as 'by complete and primary causes', but by auxiliary and proximate causes". (Cicero, 62C)

A full understanding of Chrysippus' attempted resolution of the problem of how anything can be up to us when the history of the world is such that the present chapter of the narrative is inescapable given what has come before is rendered difficult by the lack of clarity around the various kinds of causes. It is clear enough, at least in general terms, what outcome Chrysippus was aiming with respect to human action. In a famous analogy, he treats a person's character as analogous to the shape of a cylinder. It is true that the world gives us things to react to, just as a person might give the cylinder a shove. But the cylinder rolls, rather than slides, because of its specific shape (i.e. its nature). So too your decisions are your decisions in as much as the kind of person you are makes a difference to what you decide to do. Sure – you are the kind of person that you are in no small part because of what has happened to you previously. But when your decisions play a role in bringing about what you do, the Stoics say that what comes about through fate comes about through you and those actions are 'up to you' in some sense appropriate to the notion of responsibility. That sense is allegedly supplied by the distinction among the kinds of causes introduced above. Detailed scholarly work on the question of free will and determinism in Stoicism seeks to engage with our various sources and attempts to position this very different framework for thinking about causes and causation in relation to our own. A good guide to the terrain is provided by Hankinson, chapters 14 and 15 in Algra, Barnes, Mansfeld &

Schofield (1999). Bobzien (1998) is longer and perhaps more difficult for the beginning philosopher, but very authoritative.

The Stoics also discuss a notion of freedom that is rather more moral than metaphysical. This sense of freedom involves 'the power to live as you will' (Cicero, Stoic Paradoxes 5, 34). It turns out, for reasons that will be discussed below in the section on ethics, that only the Stoic wise man is truly free. All others are slaves. This notion of freedom and its relation to Kantian autonomy is discussed in Cooper (2004).

6.5 LOGIC

The scope of what the Stoics called 'logic' (logikê, i.e. knowledge of the functions of logos or reason) was very wide, including not only the analysis of argument forms, but also rhetoric, grammar, the theories of concepts, propositions, perception, and thought. Thus Stoic logikê included not only what we would call logic, but also philosophy of language and epistemology.

In philosophy of language, their most noted innovation was their theory of 'sayables' or lekta. The Stoics distinguish between the signification, the signifier and the name-bearer. Two of these are bodies: the signifier which is the utterance and the name-bearer which is what gets signified. The signification, however, is an incorporeal thing called a lekton, or 'sayable,' and it, and neither of the other two, is what is true or false (Sextus Empiricus, 33B). They define a sayable as "that which subsists in accordance with a rational impression." Rational impressions are those alterations of the commanding faculty or rational mind whose content can be exhibited in language. Presumably 'graphei Sôkratês' and 'Socrates writes' exhibit the contents of one and the same rational impression in different languages.

At first glance, this looks very like a modern theory of propositions and indeed propositions (axiômata) are one subspecies of Stoic sayables. But it would be a mistake to assimilate this sub-class of sayables too closely to modern theories of propositions. Modern theories tend to treat propositions as untensed and time-indexed. When I utter the words "It's warm in Hobart today" I express the proposition that it is warm in Hobart on 25 February 2018. That's a different proposition from the one I would express with the use of those same words tomorrow. If, as is all too often the case, it is cold tomorrow and what I say by means of the words “It is warm in Hobart today” is false, then the proposition did not change its truth value. The tenseless and time-indexed propositions we express with our words have their truth values eternally. Stoic axiômata are crucially different in this respect. The Stoic theory holds invariant the identity of the sayable corresponding to my utterances on the different occasions, but allows its truth value to change (Diog. Laert., 34E). In addition to these axiômata, the class of so-called ‘complete sayables’ included questions and commands, as well as syllogisms (Diog. Laert., 33F).

In the category of sayables called ‘incomplete,’ the Stoics included predicates and, as in the case of propositions, these are the meanings which we can express through the use of different languages. So the utterance ‘graphei’ in Greek presumably corresponds to the same incomplete sayable as ‘___ writes’ in English. Like some modern theories of predicates, these incomplete lekta are hungry for arguments or what the Stoics would call a nominative case (ptôsis, Diog. Laert., 33G). Curiously the Stoics distinguished between examples where the filling in of the subject that yields a complete sayable happens by means of a referring term (as in ‘Socrates writes’ and cases involving ostensive reference like ‘this one writes’. ‘This one writes’ was called ‘definite’, while ‘Socrates writes’ was predicative or middle – the latter in order to distinguish it from an indefinite predication like ‘someone writes’. The isolation of ostensive reference as a special case gives rise to another odd feature of the Stoic account of meanings and propositions. Standing in the presence of Socrates’ corpse, you can utter the words ‘Socrates is dead’ and your words correspond to a complete lekton (and one that is true at that time). But point to the body and say ‘This one is dead’ and the Stoics seem to have supposed the reference failed in such a way that the sayable ‘is destroyed’ (Alexander, 38F). This odd feature of sayables looms large in the Stoic response to competing accounts of modality.

The examples dealt with so far are examples of simple, complete sayables or propositions. The Stoics also developed an account of non-simple propositions. This interest in non-simple propositions and their logical relations was shared with philosophers in the Megarian or

Dialectical school. It set the philosophers of the Hellenistic period on the pathway to surpass Aristotle's progress in logic. His logic was 'a logic of terms'. To put the matter very briefly and far too crudely, Aristotle had developed an account of a limited range of kinds statements (e.g. All A are B, or Some A are B, or No A are B). His theory of the syllogism sought to systematically investigate all the ways of combining pairs of such statements and to identify the combinations where the first two (the premises) entail a third statement (the conclusion) of same sort purely as a result of the form of the premises rather than their content. Focused on the connections between predicate and subject terms in such statements, it had little to say about complex statements that had complete statements as parts. The Stoics, by contrast, made progress in what we now call propositional logic. They developed accounts of propositional negation ('it is not the case that p'), conjunction ('p and q'), disjunction ('p or q') and entered the on-going debate over the correct understanding of conditionals ('if p, then q'). Their accounts of the connectives joining simple propositions into complex ones also led them into questions about modal concepts (possibility, impossibility, necessity and contingency). One of the accounts they offer of the validity of arguments is that an argument is valid if, through the use of certain ground rules (themata), it is possible to reduce it to one of the five indemonstrable forms (Diog. Laert., 36A). These five indemonstrables are argument forms that should be familiar to anyone who has taken an introductory logic class:

if p then q; p; therefore q (modus ponens);
if p then q; not q; therefore not-p (modus tollens);
it is not the case that both p and q; p; therefore not-q;
either p or q; p; therefore not-q;
either p or q; not p; therefore q

Stoic contributions to logic and philosophy of language, as well as the backdrop of Aristotelian and Megarian views in the Hellenistic period, are thoroughly surveyed in a 100 page entry on the subject by Barnes, Bobzien and Mignucci in The Cambridge History of Hellenistic Philosophy (Algra et al, 1999). An abbreviated and more digestible version of this material by Bobzien appears in Inwood (2003).

Though these and other developments in logic are interesting in their own right, the Stoic treatment of certain problems about modality and bivalence are more significant for the shape of Stoicism as a whole. Chrysippus in particular was convinced that bivalence and the law of excluded middle apply even to contingent statements about particular future events or states of affairs. (The law of excluded middle says that for a proposition, p, and its contradictory, not-p, '(p or not-p)' is necessarily true, while bivalence insists that the truth table that defines a connective like 'or' contains only two values, true and false.) Aristotle's discussion in chapter 9 of On Interpretation of a hypothetical sea battle which either will or will not happen tomorrow has traditionally been taken to deny this. (The proper interpretation of Aristotle's position is in fact disputed by scholars, but that need not concern us here.) Aristotle had presented an argument that if it is either true or false now that there will be a sea battle tomorrow (and let us suppose for the sake of argument that it is false), then our present deliberation about whether we should go out and fight tomorrow would be pointless. After all, if it is already true now that there will be no battle, then whatever we decide, we won't fight. This kind of reasoning seems to pose a threat to the meaningfulness of deliberation and it is reasoning that proceeds simply from considerations about the nature of propositions and their truth or falsity. The Stoic Chrysippus seems to have connected this logically-motivated pathway to fatalism with the question of causal determinism (Cicero, 38G). He insisted that if there was motion without a cause, it would mean that some propositions would not be either true or false. But in fact, every proposition is either true or false. So he concluded that there is no motion without a cause.

It is one thing if our planning for tomorrow's sea battle is rendered pointless by the fact that, as it turns out, there will be adverse winds that prevent us from rowing out to fight the enemy. The rational coherence of planning is not threatened by the fact that sometimes the pre-conditions for our plans to be set in motion do not eventuate. That's just life as a human being. It is quite another if our deliberations are pointless because it is impossible that there should be a sea battle tomorrow. People who waste their breath debating what to do if 2+2=5 tomorrow seem to be irrational. After all, it is impossible that 2+2 should equal 5 – tomorrow or any day! So what then would we say if we were persuaded that all alternatives to what will actually happen in the future are similarly impossible? This would seem to pose a real threat to the rational coherence of planning.

The Stoics confronted a theory of modality (i.e. a theory of possibility and necessity) that claimed precisely this. Diodorus Cronus of the Dialectical school had argued that what is possible is limited to what either is or will be true at some point in the future (Boethius, 38C). So if we in fact don't ever get around to rowing out to fight the Megarians in a sea battle, then a sea battle with the Megarians was always impossible (and of course it made no more sense to consider how we should go about it than it would be to consider how what we should do in the event that 2+2=5). The means by which Diodorus arrived at this most unwelcome account of modality was called the Master Argument. He endorsed the claim that (1) truths about the past are necessary: it is not merely that they aren't other than they are – they can't be other than they are, for nothing has the power to change the past (Epictetus, 38A). He also claimed that (2) nothing impossible follows from what is possible. In the so-called Master Argument, he attempted to show that these two theses were incompatible with the claim that (3) there is something which is possible, but yet does not happen. The details of the Master Argument are a matter of much dispute. We know that it was alleged to show that these three propositions formed an inconsistent triad, but exactly how it did this remains uncertain. We also know that Diodorus' manner of resolving this inconsistency was to reject (3) and to define the possible as that which is or will be the case.

The Stoics felt the need to preserve the thesis that there are things which are possible but which do not happen. The same source that preserves the allegedly incompatible claims involved in the Master Argument tells us that the Stoics Cleanthes and Chrysippus did this in different ways. While Cleanthes rejected (1), the necessity of the past, Chrysippus rejected (2) that what is impossible does not follow from what is possible, using the following example: consider the conditional "if Dion is dead, then this one is dead" when ostensive reference is being made to

Dion. The antecedent is possible, since Dion will one day be dead. Hence, let us suppose it true. Then, by modus ponens, it follows that "this one is dead." However, the proposition that "this one is dead" is impossible (necessarily false), since one cannot make the requisite ostensive reference to a dead man so as to make it true that "this one [i.e. the (living) thing I'm pointing to] is dead," for a dead person isn't the same thing as what was there previously (Alex. Aph., 38F). This may appear utterly ad hoc to us, but it fits nicely with the Stoics' views on definite or deictic propositions. It once again illustrates the systematic character of Stoic philosophy.

Perhaps the most famous topic considered under the Stoic heading of logic is that of the criterion of truth and the Stoics' disputes with the skeptical New Academy about it. According to Chrysippus, the criterion of truth is the 'cognitive impression' (phantasia katalêptikê, lit. an impression that firmly grasps its object) (Diog. Laert., 40A). A criterion or canon of truth is an instrument for definitely determining that something is true, and the Hellenistic schools all provide some view on how it is that we are to measure or evaluate whether something is true or not. The Stoics' cognitive impression is an impression which (according to Zeno's definition, cf. Cicero, SVF I.59) "arises from that which is; is stamped and impressed in accordance with that very thing; and of such a kind as could not arise from what is not" (Sextus Empiricus, 40E). Recall that among the powers of the commanding faculty is the capacity to assent or withhold assent to impressions. The fact that it is always within our power to withhold assent means that if we are sufficiently disciplined, we are capable of avoiding error. In itself, it does not mean that we are capable of attaining knowledge, for there might not be any impressions that one can be confident in assenting to. The cognitive impression was supposed to fill that role: when you experience one of these, provided that you recognize it as such, you can, on its basis, assert definitely that the matter in question is true. It was initially supposed that such an impression commanded one's assent by its very nature: it "all but seizes us by the hair" and drags us to assent. But this optimistic assessment seems to have been qualified in the face of criticism by members of the Skeptical Academy – perhaps, even if there are such impressions, it is not so easy to be sure when one is experiencing one.

However, the Stoics do not maintain that the mere having of a cognitive impression constitutes knowledge (epistêmê). Indeed, not even assent to such an impression amounts to knowledge. Such assent is merely cognition or grasp (katalêpsis) of some individual fact. Real knowledge (epistêmê) requires cognition which is secure, firm and unchangeable by reason (Sextus Empiricus, 41C) – and, furthermore, worked into a systematic whole with other such cognitions (Arius Didymus, 41H). Weak and changeable assent to a cognitive impression is only an act of ignorance. It is not entirely clear where opinion or belief in general (doxa) stands in this categorization. Most Stoic sources define it as 'assent to the incognitive' (i.e. to an impression that does not firmly grasp its object) (see Sextus Empiricus, 41E) but some suggest that changeable assent to a cognitive impression might still count as opinion. There is a potential for serious confusion when we try to assimilate the Stoic view to contemporary epistemology. Modern definitions of knowledge make the agent's belief that P a necessary but not sufficient condition for knowing that P. For the Stoics, doxa (involving 'weak' assent) and knowledge are incompatible. In any event, there is an absolute distinction between the wise and the ignorant. Only the Stoic sage's assent to cognitive impressions clearly counts as knowledge for only a sage has the proper discipline always to avoid withdrawing assent, or assenting to things that one shouldn't. The Stoics call this epistemic virtue 'non-precipitancy' (aproptôsia) and it underlies their claim that the Stoic sage never makes mistakes (41D).

The Skeptics responded by denying the existence of cognitive impressions. According to Arcesilaus, "no impression arising from something true is such that an impression arising from something false could not also be just like it" (Cicero, 40D). So Arcesilaus denies that the third conjunct of the Stoic definition of the cognitive impression is ever satisfied. We can distinguish two specific tactics for denying this. First, the Skeptics point to cases of insanity. In his madness, Heracles had the impression that his children were, in fact, the children of his enemy Eurystheus and killed them. Since the impression must have been utterly convincing to him at the time at which he had it (judging by his subsequent action), it is clear from this that there can be false impressions which are indistinguishable from ones that are allegedly "stamped and impressed in accordance with that very thing" (Sextus Empiricus, 41H). Their second line of attack was to draw attention to objects which are so similar as to be indistinguishable (so that a completely accurate impression from one would be indistinguishable from one from the other). The story is related (Diog. Laert., 40F) that the Stoic philosopher Sphaerus (a student of Zeno's) was tricked into thinking that wax pomegranates were real. This was again supposed to show that there could be impressions arising from what is not [sc. a pomegranate] which are indistinguishable from a cognitive impression.

The Stoics met these arguments by first pointing out that Heracles' inability to distinguish cognitive from incognitive impressions in his madness says nothing about the capacities of normal human beings. It is no part of their thesis that just anyone can distinguish between cognitive and incognitive impressions. Their response to the second line of attack was two-fold. The first is a metaphysically motivated answer: if any two objects really were indistinguishable, they would be identical. This doctrine has come to be known as the identity of indiscernibles. They also replied that the Stoic sage would withhold assent in cases where things are too similar to be confident that one had it right (Cicero, 40I) – Sphaerus' response to his predicament was to say that he only assented to the proposition that it was 'reasonable' that what he was presented with were pomegranates (and that was true!).

In some ways, the Stoics have an easier time with skepticism abut knowledge than contemporary non-skeptics do. At bottom what the Stoics are committed to is the two-fold view that it is within our power to avoid falling into error and that there is a kind of impression which reveals to us the world as it really is and which is different from those impressions which might not so reveal the world. They are manifestly not committed to defending our ordinary intuitions about the range of knowledge: that most people in fact know most of the things that they and everyone else thinks that they know. Recall our observations about the difference between knowledge considered as a system of assents to cognitive impressions that is secure and unshakeable by reason and mere opinion – which may get matters right and may even involve assent to a cognitive impression, but still falls short of knowledge. In short, the Stoics set the bar for knowledge very high and were perfectly willing to accept that knowing was the exception, not the rule, in human affairs. The only person we can be sure has any knowledge is the Stoic sage and sages are as rare as the phoenix (Alex. Aph., 61N). Everyone else is equally ignorant. This absolute distinction between the wise and the ignorant is a consequence of the Stoic definition of knowledge as the "cognition which is secure and unchangeable by reason" (Arius Didymus, 41H). Either one's cognition is like this or it is not. By making opinion a kind of ignorance (contrast Plato, Rep V. 474a ff), they do not allow room for an intermediate state between the wise man and all the rest of us.

But even if we leave aside the question of whether we in fact know anything in the incredibly strong sense required for Stoic epistêmê, there are still some serious puzzles about the cognitive impression. The Stoics insist that the cognitive impression not only "arises from what is and is stamped and impressed in accordance" with the thing from which it arises, but also that it is "such as could not arise from that which is not." But it seems that we can imagine all kinds of situations in which we might be in a position where the sense impressions that we have are indistinguishable from ones that misrepresent the world. Thus, consider Descartes' evil demon hypothesis or its modern counterpart, the brain in a vat scenario. In the latter example it is stipulated that electrical stimulation of your brain by incredibly clever but unscrupulous scientists produces sense impressions that are indistinguishable from the ones that you are presently having. Surely here we have a demonstration that there could not be a true impression which is such that it could not arise from what is not. No sane person thinks that these skeptical hypotheses are actually true. The point is rather that if one of them were true, our sense experience would be indistinguishable from what (we take to be) our true and accurate sense impressions of real tables, chairs and fireplaces. Doesn't this show that there is no such thing as a cognitive impression?

Notes

One thing to note in passing is that skeptical scenarios like the evil demon or the brain in the vat did not seem to figure in the debate between the Stoics and Skeptics. The Skeptics press the point that at the time the dream may be completely convincing to the dreamer, even if she does not believe that the events actually transpired when she awakes (Cicero, Lucullus or Academica II, 88). They do not consider thought experiments in which all our sense experience is systematically misleading. But if we set this aside, there will still be one important difference between a clear and distinct impression that arises from a real fireplace and one that arises from the manipulation of my neurons by unscrupulous brain scientists. The first is caused by a fireplace, while the second is caused by some other means. When the Stoics say that a cognitive impression is "of such a sort as could not arise from what is not," they can be interpreted to mean simply that the true clear and distinct impression will be different from a false one. Nothing said thus far by the skeptics rules out the possibility that we have a mechanism that has potential to become sensitive to these differences. They might deny that the difference between the two is always something that can be discerned from the subject's point of view. We do not have a firmer means of knowing by virtue of which we check candidate impressions to see if they are really cognitive or not. Rather, we have the potential to increase our sensitivity to cognitive impressions when they are present.

If this is so, then the Stoics' position would be somewhat akin to externalist theories of knowledge or justification. Externalists insist that an agent might know a proposition or be justified in believing a proposition even when, nonetheless, the evidence for that belief is not subjectively available to the person. So, on one early externalist theory of knowledge, it was suggested that an agent might know a certain sort of proposition (e.g. that there is a fireplace here) if their belief that there is a fireplace here was caused by a reliable causal process (e.g. a normal visual system) – and not, e.g., by the interventions of wicked scientists fiddling with the subject's brain. Annas (1990) explores the possibilities for reading the Stoic view as a proto-externalist one. Perin (2005) considers the limitations of this reading.

So where does this leave the matter? If this is the right way to understand the definition of the Stoic cognitive impression, then it would seem that they win their argument with the Skeptics. Examples of false impressions that are subjectively indiscernible from clear and distinct, true, ones do not show that there are no cognitive impressions. However, the admission that a cognitive impression might be subjectively indistinguishable from a false impression does alter the sense in which the cognitive impression can serve as a criterion of truth. Assent to a cognitive impression will guarantee that what you assent to is true. But, because cognitive impressions can be indistinguishable from the subject's point of view from false ones, the Stoics can no longer say that even the sage can be confident that what seems to be a cognitive impression actually is one. Thus instead of automatically commanding assent, the cognitive impression (according to later Stoics) commands assent "if there is no impediment" (Sextus Empiricus, 40K), and if it has been successfully "tested" and is "irreversible" (cf. Sextus Empiricus, 69E). This means that I should only assent to what seems to me to be a cognitive impression if I have reason to believe that I'm not in a context where deceptive but convincing impressions are possible. But the Stoic sage never errs. So when will I have absolutely compelling reasons to believe that I'm not presented with a convincing but deceptive impression? For these reasons, the Pyrrhonian skeptic Sextus Empiricus argues that the Stoic sage will never assent to any impression. In practice, he will suspend judgement, just like the Skeptic does (41C). Another suggestion is that the Stoic sage hedges his bets by assenting only to the impression that it is reasonable that there is fireplace here (as Sphaerus did about the pomegranates, 40F). In this case it will also be hard to see how he differs from a skeptic who takes 'the reasonable' as his criterion (Sextus Empiricus, 69B).

6.6 ETHICS

In many ways, Aristotle's ethics provides the form for the adumbration of the ethical teaching of the Hellenistic schools. One must first provide a specification of the goal or end (telos) of living. This may have been thought to provide something like the dust jacket blurb or course description for the competing philosophical systems – which differed radically over how to give the required specification.

A bit of reflection tells us that the goal that we all have is happiness or flourishing (eudaimonia). But what is happiness? The Epicureans' answer was deceptively straightforward: the happy life is the one which is most pleasant. (But their account of what the highest pleasure consists in was not at all straightforward.) Zeno's answer was "a good flow of life" (Arius Didymus, 63A) or "living in agreement," and Cleanthes clarified that with the formulation that the end was "living in agreement with nature" (Arius Didymus, 63B). Chrysippus amplified this to (among other formulations) "living in accordance with experience of what happens by nature;" later Stoics inadvisably, in response to Academic attacks, substituted such formulations as "the rational selection of the primary things according to nature." The Stoics' specification of what happiness consists in cannot be adequately understood apart from their views about value and human psychology.

The best way into the thicket of Stoic ethics is through the question of what is good, for all parties agree that possession of what is genuinely good secures a person's happiness. The Stoics claim that whatever is good must benefit its possessor under all circumstances. But there are situations in which it is not to my benefit to be healthy or wealthy. (We may imagine that if I had money I would spend it on heroin which would not benefit me.) Thus, things like money are simply not good, in spite of how nearly everyone speaks, and the Stoics call them 'indifferents' (Diog. Laert., 58A) – i.e., neither good nor bad. The only things that are good are the characteristic excellences or virtues of human beings (or of human minds): prudence or wisdom, justice, courage and moderation, and other related qualities. These are the first two of the 'Stoic paradoxes' discussed by Cicero in his short work of that title: that only what is noble or fine or morally good (kalon) is good at all, and that the possession (and exercise) of the virtues is both necessary and sufficient for happiness. But the Stoics are not such lovers of paradox that they are willing to say that my preference for wealth over poverty in most circumstances is utterly groundless. They draw a distinction between what is good and things which have value (axia). Some indifferent things, like health or wealth, have value and therefore are to be preferred, even if they are not good, because they are typically appropriate, fitting or suitable (oikeion) for us.

Impulse, as noted above, is a movement of the soul toward an object. Though these movements are subject to the capacity for assent in fully rational creatures, impulse is present in all animate (self-moving) things from the moment of birth. The Stoics argue that the original impulse of ensouled creatures is toward what is appropriate for them, or aids in their self-preservation, and not toward what is pleasurable, as the Epicureans contend. Because the whole of the world is identical with the fully rational creature which is God, each part of it is naturally constituted so that it seeks what is appropriate or suitable to it, just as our own body parts are so constituted as to preserve both themselves and the whole of which they are parts. The Stoic doctrine of the natural attachment to what is appropriate (oikeiôsis) thus provides a foundation in nature for an objective ordering of preferences, at least on a prima facie basis. Other things being equal, it is objectively preferable to have health rather than sickness. The Stoics call things whose preferability is overridden only in very rare circumstances "things according to nature." As we mature, we discover new things which are according to our natures. As infants perhaps we only recognised that food and warmth are appropriate to us, but since humans are rational, more than these basic necessities are appropriate to us. The Greek term 'oikeion' can mean not only what is suitable, but also what is akin to oneself, standing in a natural relation of affection. Thus, my blood relatives are – or least ought to be – oikeioi. It is partly in this sense that we eventually come to the recognition – or at least ought to – that other people, insofar as they are rational, are appropriate to us. Cicero's quotation of Terence's line 'nothing human is alien to me' in the context of On Duties I.30 echoes this thought. It is not only other rational creatures that are appropriate to us, but also the perfection of our own rational natures. Because the Stoics identify the moral virtues with knowledge, and thus the perfection of our rational natures, that which is genuinely good is also most appropriate to us. So, if our moral and intellectual development goes as it should, we will progress from valuing food and warmth, to valuing social relations, to valuing moral virtue. Ideally, we'll have the recognition that the value that moral virtue has is of a different order to those things that we were naturally attracted to earlier. We then come to see that virtue is the only good.

Is that all there is to Stoic ethics? Some writers, such as Annas (1993), suppose that Stoic moral philosophy largely floats free of Stoic metaphysics, and especially from Stoic theology. Other writers, such as Cooper (1996, and 2012), suppose that Stoic moral philosophy is intimately intertwined with Stoic metaphysics. The latter reading draws our attention to the fact that the unfolding of God's providential plan is rational (and therefore beneficial) through and through, so that in some sense what will in fact happen to me in accordance with that plan must be appropriate to me, just like food, warmth, and those with whom I have intimate social relations.

When we take the rationality of the world order into consideration, we can begin to understand the Stoic formulations of the goal or end. "Living in agreement with nature" is meant to work at a variety of levels. Since my nature is such that health and wealth are appropriate to me (according to my nature), other things being equal, I ought to choose them. Hence the formulations of the end by later Stoics stress the idea that happiness consists in the rational selection of the things according to nature. But, we must bear in mind an important caveat here. Health and wealth are not the only things which are appropriate to me. So are other rational beings and it would be irrational to choose one thing which is appropriate to me without due consideration of the effect of that choice on other things which are also appropriate to me. This is why the later formulations stress that happiness consists in the rational selection of the things according to nature. But if I am faced with a choice between increasing my wealth (something which is prima facie appropriate to my nature) and preserving someone else's health (which is something appropriate to something which is appropriate to me, i.e. another rational being), which course of action is the rational one? The Stoic response is that it is the one which is ultimately both natural and rational: that is, the one that, so far as I can tell from my experience with what happens in the course of nature (see Chrysippus' formula for the end cited above, 63B), is most in agreement with the unfolding of nature's rational and providential plan. Living in agreement with nature in this sense can even demand that I select things which are not typically appropriate to my nature at all – when that nature is considered in isolation from these particular circumstances. Here Chrysippus' remark about what his foot would will if it were conscious is apposite.

As long as the future is uncertain to me I always hold to those things which are better adapted to obtaining the things in accordance with nature; for God himself has made me disposed to select these. But if I actually knew that I was fated now to be ill, I would even have an impulse to be ill. For my foot too, if it had intelligence, would have an impulse to get muddy. (Epictetus, 58J)

We too, as rational parts of rational nature, ought to choose in accordance with what will in fact happen (provided we can know what that will be, which we rarely can – we are not gods; outcomes are uncertain to us) since this is wholly good and rational: when we cannot know the outcome, we ought to choose in accordance with what is typically or usually nature's purpose, as we can see from experience of what usually does happen in the course of nature. In extreme circumstances, however, a choice, for example, to end our lives by suicide can be in agreement with nature.

So far the emphasis has been on just one component of the Stoic formulation of the goal or end of life: it is the "rational selection of the things according to nature." The other thing that needs to be stressed is that it is rational selection – not the attainment of – these things which constitutes happiness. (The Stoics mark the distinction between the way we ought to opt for health as opposed to virtue by saying that I select (eklegomai) the preferred indifferent but I choose (hairoûmai) the virtuous action.) Even though the things according to nature have a kind of value (axia) which grounds the rationality of preferring them (other things being equal), this kind of value is still not goodness. From the point of view of happiness, the things according to nature are still indifferent. What matters for our happiness is whether we select them rationally and, as it turns out, this means selecting them in accordance with the virtuous way of regarding them (and virtuous action itself).

Surely one motive for this is the rejection of even the limited role that external goods and fortune play in Aristotelian ethics. According to the Peripatetics, the happy life is one in which one exercises one's moral and theoretical virtues. But one can't exercise a moral virtue like liberality (Nic. Eth. IV.1) without having some, even considerable, money. The Stoics, by contrast, claim that so long as I order (and express) my preferences in accordance with my nature and universal nature, I will be virtuous and happy, even if I do not actually get the things I prefer. Though these things are typically appropriate to me, rational choice is even more appropriate or akin to me, and so long as I have that, then I have perfected my nature. The perfection of one's rational nature is the condition of being virtuous and it is exercising this, and this alone, which is good. Since possession of that which is good is sufficient for happiness, virtuous agents are happy even if they do not attain the preferred indifferents they select.

One is tempted to think that this is simply a misuse of the word 'happiness' (or would be, if the Stoics had been speaking English). We are inclined to think (and a Greek talking about eudaimonia would arguably be similarly inclined) that happiness has something to do with getting what you want and not merely ordering one's wants rationally regardless of whether they are satisfied. People are also frequently tempted to assimilate the Stoics' position to one (increasingly contested) interpretation of Kant's moral philosophy. On this reading, acting with the right motive is the only thing that is good – but being good in this sense has nothing whatsoever to do with happiness.

With respect to the first point, the Stoic sage typically selects the preferred indifferents and selects them in light of her knowledge of how the world works. There will be times when the circumstances make it rational for her to select something that is (generally speaking) contrary to her nature (e.g., cutting off one's own hand in order to thwart a tyrant). But these circumstances will be rare and the sage will not be oppressed by the additional false beliefs that this act of self-mutilation is a genuinely bad thing: only vice is genuinely bad. For the most part, her knowledge of nature and other people will mean that she attains the things that she selects. Her conditional positive attitude toward them will mean that when circumstances do conspire to bring it about that the object of her selection is not secured, she doesn't care. She only preferred to be wealthy if it was fated for her to be wealthy. These reflections illustrate the way in which the virtuous person is self-sufficient (autarkês) and this seems to be an important component of our intuitive idea of happiness. The person who is genuinely happy lacks nothing and enjoys a kind of independence from the vagaries of fortune. To this extent at least, the Stoics are not just using the word 'happiness' for a condition that has nothing at all to do with what we typically mean by it. With respect to the second point, the Stoic sage will never find herself in a situation where she acts contrary to what Kant calls inclination or desire. The only thing she unconditionally wants is to live virtuously. Anything that she conditionally prefers is always subordinate to her conception of the genuine good. Thus, there is no room for a conflict between duty and happiness where the latter is thought of solely in terms of the satisfaction of our desires. Cicero provides an engaging, if not altogether rigorous, discussion of the question of whether virtue is sufficient for happiness in Tusculan Disputations, book V.

How do these general considerations about the goal of living translate into an evaluation of actions? When I perform an action that accords with my nature and for which a good reason can be given, then I perform what the Stoics call (LS) a 'proper function' (kathêkon, Arius Didymus, 59B) – something that it "falls to me" to do. It is important to note that non-rational animals and plants perform proper functions as well (Diog. Laert., 59C). This shows how much importance is placed upon the idea of what accords with one's nature or, in another formulation, "activity which is consequential upon a thing's nature." It also shows the gap between proper functions and morally right actions, for the Stoics, like most contemporary philosophers, think that animals cannot act morally or immorally – let alone plants.

Most proper functions are directed toward securing things which are appropriate to nature. Thus, if I take good care of my body, then this is a proper function. The Stoics divide proper functions into those which do not depend upon circumstances and those that do. Taking care of one's health is among the former, while mutilating oneself is among the latter

(Diog. Laert., 59E). It appears that this is an attempt to work out a set of prima facie duties based upon our natures. Other things being equal, looking after one's health is a course of action which accords with one's nature and thus is one for which a good reason can be given. However, there are circumstances in which a better reason can be given for mutilating oneself – for instance, if this is the only way you can prevent Fagin from compelling you to steal for him.

Since both ordinary people and Stoic wise men look after their health except in very extraordinary circumstances, both the sage and the ordinary person perform proper functions. A proper function becomes a fully correct action (katorthôma) only when it is perfected as an action of the specific kind to which it belongs, and so is done virtuously. In the tradition of Socratic moral theory, the Stoics regard virtues like courage and justice, and so on, as knowledge or science within the soul about how to live. Thus a specific virtue like moderation is defined as "the science (epistêmê) of what is to be chosen and what is to be avoided and what is neither of these" (Arius Didymus, 61H). More broadly, virtue is "an expertise (technê) concerned with the whole of life" (Arius Didymus, 61G). Like other forms of knowledge, virtues are characters of the soul's commanding faculty which are firm and unchangeable. The other similarity with Socratic ethics is that the Stoics think that the virtues are really just one state of soul (Plutarch , 61B, C; Arius Didymus, 61D). No one can be moderate without also being just, courageous and prudent as well – moreover, "anyone who does any action in accordance with one does so in accordance with them all" (Plutarch, 61F). When someone who has any virtue, and therefore all the virtues, performs any proper function, he performs it in accordance with virtue or virtuously (i.e. with all the virtues) and this transforms it into a right action or a perfect function. The connection here between a perfect function and a virtuous one is almost analytic in Greek ethical theorizing. Virtues just are those features which make a thing a good thing of its kind or allow it to perform its function well. So, actions done in accordance with virtue are actions which are done well. The Stoics draw the conclusion from this that the wise (and therefore virtuous) person does everything within the scope of moral action well (Arius Didymus, 61G). This makes it seem far less strange than it might at first appear to say that virtue is sufficient for happiness. Furthermore, because virtue is a kind of knowledge and there is no cognitive state between knowledge and ignorance, those who are not wise do everything equally badly. Strictly speaking, there is no such thing as moral progress for the Stoics (if that means progress within morality), and they give the charming illustration of drowning to make their point: a person an arm's length from the surface is drowning every bit as surely as one who is five hundred fathoms down (Plutarch, 61T). Of course, as the analogy also suggests, it is possible to be closer or farther from finally being able to perform proper functions in this perfected way. In that sense, progress is possible.

We are finally in a position to understand and evaluate the Stoic view on emotions, since it is a consequence of their views on the soul and the good. It is perhaps more accurate to call it the Stoic view of the passions, though this is a somewhat dated term. The passions or pathê are literally 'things which one undergoes' and are to be contrasted with actions or things that one does. Thus, the view that one should be 'apathetic,' in its original Hellenistic sense, is not the view that you shouldn't care about anything, but rather the view that you should not be psychologically subject to anything – manipulated and moved by it, rather than yourself being actively and positively in command of your reactions and responses to things as they occur or are in prospect. It connotes a kind of complete self-sufficiency. The Stoics distinguish two primary passions: appetite and fear. These arise in relation to what appears to us to be good or bad. They are associated with two other passions: pleasure and distress. These result when we get or fail to avoid the objects of the first two passions. What distinguishes these states of soul from normal impulses is that they are "excessive impulses which are disobedient to reason" (Arius Didymus, 65A). Part of what this means is that one's fear of dogs may not go away with the rational recognition that this blind, 16 year old, 3 legged Yorkshire terrier poses no threat to you. But this is not all. The Stoics call a passion like distress a fresh opinion that something bad is present (Andronicus, 65B): you may have been excitedly delighted when you first saw you'd won the race, but after a while, when the impression of the victory is no longer fresh, you may calm down. Recall that opinion is assent to a false impression. Given the Stoics' view about good and bad, as against merely indifferent things, the only time that one should assent to the impression that something bad is present is when there is something which might threaten one's virtue, for this and this alone is good. Thus all passions involve an element of false value-judgement. But these are false judgements which are inseparable from physiological changes in the pneuma which constitutes one's commanding faculty. The Stoics describe these changes as shrinkings (like fear) or swellings (like delight), and part of the reason that they locate the commanding faculty in the heart (rather than the head, as Plato in the Timaeus and many medical writers did) is that this seems to be where the physical sensations which accompany passions like fear are manifested. Taking note of this point of physiology is surely necessary to give their theory any plausibility. From the inside a value-judgement – even one like "this impending dog bite will be bad" – might often just not feel like such an emotional state as fear. But when the judgement is vivid and so the commanding faculty is undergoing such a change, one can readily enough see that the characteristic sensations might inexorably accompany the judgement.

Another obvious objection to the Stoic theory is that someone who fears, say pigeons, may not think that they are dangerous. We say that she knows rationally that pigeons are harmless but that she has an irrational fear. It might be thought that in such a case, the judgement which the Stoics think is essential to the passion is missing. Here they resort to the idea that a passion is a fluttering of the commanding faculty. At one instant my commanding faculty judges (rightly) that this pigeon is not dangerous, but an instant later assents to the impression that it is and from this assent flows the excessive impulse away from the pigeon which is my fear. This switch of assent occurs repeatedly and rapidly so that it appears that one has the fear without the requisite judgement but in fact you are making it and taking it back during the time you undergo the passion (Plutarch, 65G).

It is important to bear in mind that the Stoics do not think that all impulses are to be done away with. What distinguishes normal impulses or desires from passions is the idea that the latter are excessive and irrational. Galen provides a nice illustration of the difference (65J). Suppose I want to run, or, in Stoic terminology, I have an impulse to run. If I go running down a sharp incline I may be unable to stop or change direction in response to a new impulse. My running is excessive in relation to my initial impulse. Passions are distinguished from normal impulses in much the same way: they have a kind of momentum which carries one beyond the dictates of reason. If, for instance, you are consumed with lust (a passion falling under appetite), you might not do what under other circumstances you yourself would judge to be the sensible thing.

Even in antiquity the Stoics were ridiculed for their views on the passions. Some critics called them the men of stone. But this is not entirely fair, for the Stoics allow that the sage will experience what they call the good feelings (eupatheiai, Diog. Laert. 65F). These include joy, watchfulness and wishing and are distinguished from their negative counterparts (pleasure, fear and appetite) in being well-reasoned and not excessive. Naturally there is no positive counterpart to distress. The species under wishing include kindness, generosity and warmth. A good feeling like kindness is a moderate and reasonable stretching or expansion of the soul presumably prompted by the correct judgement that other rational beings are appropriate to oneself.

Criticisms of the Stoic theory of the passions in antiquity focused on two issues. The first was whether the passions were, in fact, activities of the rational soul. The medical writer and philosopher Galen defended the Platonic account of emotions as a product of an irrational part of the soul. Posidonius, a 1st c. BCE Stoic, also criticised Chrysippus on the psychology of emotions, and developed a position that recognized the influence in the mind of something like Plato’s irrational soul-parts. The other opposition to the Stoic doctrine came from philosophers in the Aristotelian tradition. They, like the Stoics, made judgement a component in emotions. But they argued that the happy life required the moderation of the passions, not their complete extinction. Cicero’s Tusculan Disputations, books III and IV take up the question of whether it is possible and desirable to rid oneself of the emotions.

6.7 INFLUENCE

6.1 On Greek culture and politics

The ordinary Greek in the street may have had little idea of the views of Plato or Aristotle. The founder of the Stoic school, however, had a statue raised to him in Athens at public expense, the inscription on which read, in part:

Whereas Zeno of Citium, son of Mnaseas, has for many years been devoted to philosophy in the city and has continued to be a man of worth in all other respects, exhorting to virtue and temperance those of the youth who came to him to be taught, directing them to what is best, affording to all in his own conduct a pattern for imitation in perfect consistency with his teaching … (Diog. Laert. 7.10–11, tr. Hicks)

Of course the citizens of Athens couldn't have honoured Zeno for a life lived in consistency with his philosophical principles unless the content of those principles was known to the general public. Since the Stoics gathered, discussed and taught philosophy in a public place, the general import of their philosophy was widely known. Stoicism became a "popular philosophy" in a way that neither Platonism nor Aristotelianism ever did. In part this is because Stoicism, like its rival Epicureanism, addressed the questions that most people are concerned with in very direct and practical ways. It tells you how you should regard death, suffering, great wealth, poverty, power over others and slavery. In the political and social context of the Hellenistic period (where a person could move between these extremes in very short order) Stoicism provided a psychological fortress against bad fortune.

At the political level, the Antigonid dynasty (which ruled Greece and Macedon from shortly after the death of Alexander to 168 BCE) had connections with the Stoic philosophers. Antigonus Gonatas was alleged to have been a pupil of Zeno of Citium. He requested that Zeno serve as the tutor to his son, Demetrius, but Zeno excused himself on the ground that he was too old for the job. The man he sent instead, Persaeus, was deeply involved in affairs at court and, according to some sources, died in battle at Corinth in the service of Antigonus. Another Hellenistic strong-man, Cleomenes of Sparta, had the Stoic philosopher Sphaerus as one of his advisors. The reforms instituted in Sparta (including the extension of citizenship to foreigners and the redistribution of land) were seen by some as a Stoic social reform, though it is less clear that it was anything other than an instrument of power for Cleomenes. (For one view, see Erskine 1990 chapter 6; for a more cynical view see Green 1990, p. 248 ff.)

1. On "Middle Stoicism"

Middle Stoicism is the term used to encompass the work of later Stoic philosophers including Antipater of Tarsus (d. 130/129 BCE), Panaetius (d. 110/09 BCE), and Posidonius (d. ~45 BCE). Earlier scholarship on Middle Stoicism tended to accentuate the degree of discontinuity between it and the "Old Stoa". It is certainly true that there was evolution in Stoic ideas with these philosophers and disagreements with earlier Stoics. Thus, for instance, Antipater was much more positive about marriage and family than Chrysippus was. We can, in many cases, attribute the Middle Stoa's divergence from the Old to a desire to amalgamate what these writers took to be correct in other philosophical schools. In particular, these Stoics looked to Platonism and especially to Plato's dialogue the Timaeus; cf. Reydams-Schils (1999). Panaetius denied the periodic conflagration posited by earlier Stoic philosophers (Van Staaten, fr. 65). Posidonius, though he is wrongly reported by Galen to have returned to Plato's tri-partite soul and to have rejected Chrysippus' purely intellectualist theory of emotion (on this interpretation, see Sorabji 2000, 94ff), he did think it necessary to acknowledge non-rational movements in the human soul corresponding to Plato's appetite and spirit (see Cooper 1999, 449–84). In spite of these differences, however, in many other ways the Middle Stoics remained, well, Stoics.

Our evidence for the views of the philosophers of the Middle Stoa is relatively fragmentary. The testimonia for Antipater were included in volume 3 of von Arnim (1903–05). For Panaetius, see van Staaten (1962) and for Posidonius, see Edelstein and Kidd (1972). Panaetius hovers in the background of one of the most influential books in moral philosophy up through the late 19th century: Cicero's On Duties or De Officiis. In one of his letters to his friend Atticus (XVI. 11.4) Cicero says that he based the first two books of his work on Panaetius' treatise of the same name. It is perhaps on this basis that some interpreters have taken Middle Stoic moral philosophy to be more "practical" than that of the Old Stoa, for On Duties concentrates on identifying proper functions in a context where it is clear we are not talking about the infallible Stoic sage. But Sedley (in Inwood, 2003) correctly points out that any work on proper functions would have just such a focus. Our evidence may constitute an unrepresentative sample of Panaetius' work in moral philosophy.

2. On Roman political life

In 155 BCE Athens sent a delegation of three philosophers (Stoic, Academic skeptic, and Peripatetic) on an embassy to Rome. Their teachings caused a sensation among the educated. The Skeptic Carneades addressed a crowd of thousands on one day and argued that justice was a genuine good in its own right. The next day he argued against the proposition that it was in an agent's interest to be just in terms every bit as convincing. This dazzling display of dialectical skill, together with the deep seated suspicion of philosophical culture, generated a conservative backlash against all Greek philosophers led by Cato the Elder (234–149 BCE). By 86 BCE, however, Rome was ready to receive Greek philosophy with open arms.

It was natural that an ambitious and well off Roman like Cicero (106–43 BCE) should go and study at the philosophical schools in Athens and return to popularise Greek philosophy for his less cosmopolitan countrymen. Epicureanism tended to be favored in the ranks in Rome's military, while Stoicism appealed more to members of the Senate and other political movers and shakers. Several Roman political figures associated with Julius Caesar and the end of the Roman Republic had assorted philosophical connections. Those associated with Stoicism include Cato the Younger (95–46 BCE) and Marcus Junius Brutus (85–42). Brutus' fellow assassin, and brother-in-law Gaius Cassius Longinus (85–42) professed Epicureanism.(See Sedley 1997 for an examination of their actions in light of their philosophical allegiances.) Posidonius was known to Julius Caesar's sometimes-ally, sometimes-adversary, Pompey (106–48). Pompey visited Posidonius in Rhodes during his campaigns in

66 and 62 BCE. Gaius Octavius (who became Caesar Augustus) had a Stoic tutor, Athenodoros Calvus.

3. On Roman philosophers

In contrast to the fragmentary evidence that we possess for the philosophers of the Old and Middle Stoa, we have substantial writings from a number of Roman Stoic philosophers. Two of them wrote in Greek, Epictetus (circa 55–155 CE) and the Roman emperor Marcus Aurelius (121–180 CE), while the third wrote in Latin, Lucius Annaeus Seneca (4 BCE–65 CE). Other Roman Stoics whose works have not been so well preserved include Musonius Rufus (c. 25–90 CE) and Hierocles the Stoic (c. 150 CE – not to be confused with the 5th century Neoplatonist of the same name who wrote a commentary on the ps-Pythagorean 'Golden Verses').

In spite of the fact that we have more evidence for these Roman Stoics, scholarship has treated these philosophers – and particularly Seneca – primarily as sources of evidence for early Stoicism. Happily this situation has changed significantly over the last decade so that Marcus Aurelius and Seneca are being read as thinkers in their own right. (Epictetus has always been treated somewhat more seriously.) In what follows I will simply gesture toward some of excellent work being done on Roman Stoicism. The detailed work of scholarship has shown the dangers of generalising about Roman Stoicism in opposition to the original Stoic philosophy of Zeno, Cleanthes and Chrysippus. In spite of this, it is perhaps not too rash for the purposes of this encyclopedia entry to say the following: (1) Epictetus, Seneca and other Roman Stoics are less interested in what we might call the metaphysical theory of the mind or soul in relation to the body and more interested in the psychological and moral category of the self. This is not to say that the Roman Stoics retreat from the earlier Stoic materialism. It is rather that they were more interested in notions that we might call self-hood or personality. See Gill (2006) and, more broadly, Sorabji (2006). (2) The Roman Stoics may or may not have resiled from the earlier absolute distinction between the sage (who alone is wise, virtuous and happy) and everyone else (who are all equally ignorant, equally vicious, and equally unhappy). But, in any case, the writings that we have from them show much more interest in the problems that confront the person who is still making progress toward wisdom. The central theme of Reydams-Schils (2005) is that their notion of the self provides the bridge between the ideal of the Stoic sage and the actual world of less-than-ideal communities and families. (3) While it seems unlikely that any of the Roman Stoics retreated from the causal determinism and compatibilism of the Old Stoa, they were much more interested in a psychological notion of freedom. This ideal of freedom stands in opposition, not to universal causation, but rather to a self-imposed slavery that is the result of taking external things to be genuine goods. See Stephens (2007). (4) A significant portion of the writings of the Roman Stoics concern how one might move from the abstract recognition that, for instance, anger is a mistake to the condition of being immune to anger. Recent scholarship has considered these techniques, often under the label of 'spiritual exercises' For an example, see the careful reading of Marcus Aurelius' Meditations in Hadot (1998). Against the assimilation of Stoic techniques of emotional 'therapy' to Christian 'spiritual exercises', see Cooper 2012.)

4. On Christianity

Christian writers were certainly receptive to some of the elements of Stoicism. There exists an inauthentic correspondence between St Paul and Seneca included in the Apocrypha. This forgery is a very ancient one, since it was referred to in both Jerome (de Viris Illustribus 12) and Augustine (Epistle 153.4). But the fact that the letters were not written by Paul or by Seneca does not mean that Paul was unaware of Stoic philosophy, nor that his own thought may not be understood in relation to Stoic naturalism. See Engberg-Pedersen 2000. The tradition of theories of natural law in ethics seems to stem directly from Stoicism. (Compare Cicero, de Legibus I, 18 with later writers like Aquinas in Summa Theologica II, 2, q. 94.) Augustine, alas, chose to follow the Stoics rather than the Platonists (his usual allies among the philosophers) on the question of animals' membership in the moral community (City of God 1.20). Sorabji (2000), part IV argues that the Stoic idea of freedom from the passions was adapted and transmuted into the idea of seven deadly sins by Evagrius. In general, see Colish (1985) for the presence of Stoicism in Latin writers through the sixth century.

The influence of Stoicism on Medieval thought has been considered by Verbeke (1983). In general, the handling of Stoic ideas in the context of Christian orthodoxy required a certain delicacy. While it was agreed by nearly all that God was not a material being, the state of the human soul was a more controversial matter. In general, orthodoxy evolved away from materialist anthropology of the sort found in Tertullian to the immaterialist notion of the soul that present-day Christians take for granted. Medieval Christians felt it necessary to reject what they called Stoic fatalism, but notions of conscience and natural law had clear connections with Stoic thought.

5. On Renaissance and early modern philosophy

The late 16th and early 17th centuries saw efforts to form a systematic synthesis of Christianity and Stoicism. The most important figure in the Neo-Stoic movement was Justus Lipsius (1547–1606). Lipsius has his own separate entry in the Stanford Encyclopedia, so I will not discuss him further. See also Cooper (2004). The influence of the Hellenistic schools generally on early modern philosophy is the theme of the essays collected in Miller and Inwood (2003). See also Osler (1991) and Strange & Zubek (2004).

6. On modern experiments in living

Academic interest in Stoicism in the late 20th and early 21st century has been matched by interest in the therapeutic aspects of the Stoic way of life by those who are not specialists in the history of philosophy. There seem to be strong affinities between the central role that Stoicism accords to judgement and the techniques of Cognitive Behavioral Therapy or CBT. Among the most prominent (and historically grounded) is Stoicism Today which runs events such as Live Like a Stoic Week. Another modern application of Stoicism is in the field of military ethics. See Sherman (2005).

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What do you know the Sources of our information on the Stoics?

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2. Discuss the Philosophy and life.

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3. What do you know about the Physical Theory?

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4. Discuss the Logic.

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6.8 LET US SUM UP

Stoicism originated as a Hellenistic philosophy, founded in Athens by Zeno of Citium (modern day Cyprus), c. 300 B.C.E. It was influenced by Socrates and the Cynics, and it engaged in vigorous debates with the Skeptics, the Academics, and the Epicureans. The name comes from the Stoa Poikile, or painted porch, an open market in Athens where the original Stoics used to meet and teach philosophy. Stoicism moved to Rome where it flourished during the period of the Empire, alternatively being persecuted by Emperors who disliked it (for example, Vespasian and Domitian) and openly embraced by Emperors who attempted to live by it (most prominently Marcus Aurelius). It influenced Christianity, as well as a number of major philosophical figures throughout the ages (for example, Thomas More, Descartes, Spinoza), and in the early 21st century saw a revival as a practical philosophy associated with Cognitive Behavioral Therapy and similar approaches. Stoicism is a type of eudaimonic virtue ethics, asserting that the practice of virtue is both necessary and sufficient to achieve happiness (in the eudaimonic sense). However, the Stoics also recognized the existence of "indifferents" (to eudaimonia) that could nevertheless be preferred (for example, health, wealth, education) or dispreferred (for example, sickness, poverty, ignorance), because they had (respectively, positive or negative) planning value with respect to the ability to practice virtue. Stoicism was very much a philosophy meant to be applied to everyday living, focused on ethics (understood as the study of how to live one's life), which was in turn informed by what the Stoics called "physics" (nowadays, a combination of natural science and metaphysics) and what they called "logic" (a combination of modern logic, epistemology, philosophy of language, and cognitive science).

6.9 KEY WORDS

Stoicism: Stoicism originated as a Hellenistic philosophy, founded in Athens by Zeno of Citium (modern day Cyprus), c. 300 B.C.E. It was influenced by Socrates and the Cynics, and it engaged in vigorous debates with the Skeptics, the Academics, and the Epicureans.

6.10 QUESTIONS FOR REVIEW

1. What do you know the Ethics?
2. Describe the Influence

6.11 SUGGESTED READINGS AND REFERENCES

- Long, A. A. and Sedley, D. N., 1987, The Hellenistic Philosophers 2 vols. Cambridge: Cambridge University Press [Vol. 2 contains an extensive bibliography of scholarly books and articles.]
- Inwood, B. and Gerson, L., 1997, Hellenistic Philosophy 2nd ed. Indianapolis: Hackett Publishing 1997. [This volume is cheaper than Long and Sedley, but it lacks the valuable commentary that LS provide. On the other hand, Inwood and Gerson give you more texts on Pyrrhonism.]

- Dufour, Richard, 2004, Chrysippe. Oeuvre philosophique, 2 volumes, Paris: Les Belles Lettres
- Nickel, R., 2009, Stoa und Stoiker. Auswahl der Fragmente und Zeugnisse, 2 volumes, Dusseldorf: Artemis und Winkler
- von Arnim, H., 1903–5 Stoicorum Veterum Fragmenta Leipzig: Teubner (Volume 4 indexes, 1924).

6.12 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 6.2
2. See Section 6.3
3. See Section 6.4
4. See Section 6.5

UNIT 7: MODAL LOGIC

STRUCTURE

7.0 OBJECTIVES

After this unit, we can understand

- 7.1 What is Modal Logic?
- 7.2 Modal Logics
- 7.3 Deontic Logics
- 7.4 Temporal Logics
- 7.5 Conditional Logics
- 7.6 Possible Worlds Semantics

Notes

7.1 INTRODUCTION

A modal is an expression (like ‘necessarily’ or ‘possibly’) that is used to qualify the truth of a judgement. Modal logic is, strictly speaking, the study of the deductive behavior of the expressions ‘it is necessary that’ and ‘it is possible that’. However, the term ‘modal logic’ may be used more broadly for a family of related systems. These include logics for belief, for tense and other temporal expressions, for the deontic (moral) expressions such as ‘it is obligatory that’ and ‘it is permitted that’, and many others. An understanding of modal logic is particularly valuable in the formal analysis of philosophical argument, where expressions from the modal family are both common and confusing. Modal logic also has important applications in computer science.

7.2 WHAT IS MODAL LOGIC?

Narrowly construed, modal logic studies reasoning that involves the use of the expressions ‘necessarily’ and ‘possibly’. However, the term ‘modal logic’ is used more broadly to cover a family of logics with similar rules and a variety of different symbols.

A list describing the best known of these logics follows.

Logic	**Symbols**	**Expressions Symbolized**
Modal Logic	ϒ⌋	It is necessary that …
	◊◊	It is possible that …
Deontic Logic	OO	It is obligatory that …
	PP	It is permitted that …
	FF	It is forbidden that …
Temporal Logic	GG	It will always be the case that …
	FF	It will be the case that …
	HH	It has always been the case that …
	PP	It was the case that …
Doxastic Logic	BxBx	xx believes that …

7.3 MODAL LOGICS

The most familiar logics in the modal family are constructed from a weak logic called KK (after Saul Kripke). Under the narrow reading, modal logic concerns necessity and possibility. A variety of different systems may be developed for such logics using KK as a foundation. The symbols of KK include '∼∼' for 'not', '→→' for 'if…then', and 'ϒ⌋' for the modal operator 'it is necessary that'. (The connectives '&&', '∨∨', and '↔↔' may be defined from '∼∼' and '→→' as is done in propositional logic.) KK results from adding the following to the principles of propositional logic.

Necessitation Rule: If AA is a theorem of KK, then so is□ϒA⌋A.

Distribution Axiom: ϒ(A→B)→(ϒA→ϒB)⌋(A→B)→(⌋A→⌋B).

(In these principles we use 'AA' and 'BB' as metavariables ranging over formulas of the language.) According to the Necessitation Rule, any theorem of logic is necessary. The Distribution Axiom says that if it is necessary that if AA then BB, then if necessarily AA, then necessarily BB.

The operator ⊇⊇ (for 'possibly') can be defined from ϒ⌋ by letting ⊇A=∼ϒ∼A◊A=∼⌋∼A. In KK, the operators ϒ⌋ and ⊇⊇ behave very much like the quantifiers ∀∀ (all) and ∃∃ (some). For example, the definition of ⊇⊇ from ϒ⌋ mirrors the equivalence of ∀xA∀xA with ∼∃x∼A∼∃x∼A in predicate logic. Furthermore, ϒ(A&B)⌋(A&B) entails ϒA&ϒB⌋A&⌋B and vice versa; while ϒA∨ϒB⌋A∨⌋B entails ϒ(A∨B)⌋(A∨B), but not vice versa. This reflects the patterns exhibited by the universal quantifier: ∀x(A&B)∀x(A&B) entails ∀xA&∀xB∀xA&∀xB and vice versa, while ∀xA∨∀xB∀xA∨∀xB entails ∀x(A∨B)∀x(A∨B) but not vice versa. Similar parallels between ◊◊ and ∃∃ can be drawn. The basis for this correspondence between the modal operators and the quantifiers will emerge more clearly in the section on Possible Worlds Semantics.

The system KK is too weak to provide an adequate account of necessity. The following axiom is not provable in KK, but it is clearly desirable.

ϒA→A(M)(M)⌋A→A

(M)(M) claims that whatever is necessary is the case. Notice that (M)(M) would be incorrect were ϒ⌋ to be read 'it ought to be that', or 'it was the case that'. So the presence of axiom (M)(M) distinguishes logics for necessity from other logics in the modal family. A basic modal logic MM results from adding (M)(M) to KK. (Some authors call this system TT.)

Many logicians believe that MM is still too weak to correctly formalize the logic of necessity and possibility. They recommend further axioms to govern the iteration, or repetition of modal operators. Here are two of the most famous iteration axioms:

ϒA→ϒϒA(4)(4)⌋A→⌋⌋A

◊A→ϒ◊A(5)(5)◊A→⌋◊A

S4S4 is the system that results from adding (4) to MM. Similarly S5S5 is MM plus (5). In S4S4, the sentence ϒϒA⌋⌋A is equivalent to ϒA⌋A. As a result, any string of boxes may be replaced by a single box, and the same goes for strings of diamonds. This amounts to the idea that iteration of the modal operators is superfluous. Saying that AA is necessarily necessary is considered a uselessly long-winded way of saying that AA is necessary. The system S5S5 has even stronger principles for simplifying strings of modal operators. In S4S4, a string of operators of the same kind can be replaced for that operator; in S5S5, strings containing both boxes and diamonds are equivalent to the last operator in the string. So, for example, saying that it is possible that AA is necessary is the same as saying that AA is necessary. A summary of these features of S4S4 and S5S5 follows.

ϒϒ…ϒ=ϒ and ⊇⊇…⊇=⊇(S4)(S4)⌋⌋□⌋=⌋ and ⊇⊇□⊇=⊇

00□ϒ=ϒ and 00 □⊇=⊇, where each 0 is either ϒ or ◊(S5)(S5)00…⌋=⌋ and 00…◊=◊, where each 0 is either ⌋ or ◊

One could engage in endless argument over the correctness or incorrectness of these and other iteration principles for ϒ⌋ and ◊◊. The controversy can be partly resolved by recognizing that the words 'necessarily' and 'possibly', have many different uses. So the acceptability of axioms for modal logic depends on which of these uses we have in mind. For this reason, there is no one modal logic, but rather a whole family of systems built around MM. The relationship between these systems is diagrammed in Section 8, and their application to different uses of ‘necessarily’ and ‘possibly’ can be more deeply understood by studying their possible world semantics in Section 6.

The system BB (for the logician Brouwer) is formed by adding axiom (B)(B) to MM.

A→ϒ◊A(B)(B)A→⌋◊A

It is interesting to note that S5S5 can be formulated equivalently by adding (B)(B) to S4S4. The axiom (B)(B) raises an important point about the interpretation of modal formulas. (B)(B) says that if AA is the case, then AA is necessarily possible. One might argue that (B)(B) should always be adopted in any modal logic, for surely if AA is the case, then it is necessary that AA is possible. However, there is a problem with this claim that can be exposed by noting that ◊ϒA→A◊⌋A→A is provable from (B)(B). So ◊ϒA→A◊⌋A→A should be acceptable if (B)(B) is. However, ◊ϒA→A◊⌋A→A says that if AA is possibly necessary, then AA is the case, and this is far from obvious. Why does (B)(B) seem obvious, while one of the things it entails seems not obvious at all? The answer is that there is a dangerous ambiguity in the English interpretation of A→ϒ◊AA→⌋◊A. We often use the expression ‘If AA then necessarily BB’ to express that the conditional ‘if AA then BB’ is necessary. This interpretation corresponds to ϒ(A→B)⌋(A→B). On other occasions, we mean that if AA, then BB is necessary: A→ϒBA→⌋B. In English, ‘necessarily’ is an adverb, and since adverbs are usually placed near verbs, we have no natural way to indicate whether the modal operator applies to the whole conditional, or to its consequent. For these reasons, there is a tendency to confuse (B):A→ϒ◊A(B):A→⌋◊A with ϒ(A→◊A)⌋(A→◊A).

But ϒ(A→◊A)⌋(A→◊A) is not the same as (B)(B), for ϒ(A→◊A)⌋(A→◊A) is already a theorem of MM, and (B)(B) is not. One must take special care that our positive reaction to ϒ(A→◊A)⌋(A→◊A) does not infect our evaluation of (B)(B). One simple way to protect ourselves is to formulate BB in an equivalent way using the axiom: ◊ϒA→A◊⌋A→A, where these ambiguities of scope do not arise.

3. Deontic Logics

Deontic logics introduce the primitive symbol OO for 'it is obligatory that', from which symbols PP for 'it is permitted that' and FF for 'it is forbidden that' are defined: PA=∼O∼APA=∼O∼A and FA=O∼AFA=O∼A. The deontic analog of the modal axiom (M):OA→A(M):OA→A is clearly not appropriate for deontic logic. (Unfortunately, what ought to be is not always the case.) However, a basic system DD of deontic logic can be constructed by adding the weaker axiom (D)(D) to KK.

OA→PA(D)(D)OA→PA

Axiom (D)(D) guarantees the consistency of the system of obligations by insisting that when AA is obligatory, AA is permissible. A system which obligates us to bring about AA, but doesn't permit us to do so, puts us in an inescapable bind. Although some will argue that such conflicts of obligation are at least possible, most deontic logicians accept (D)(D).

O(OA→A)O(OA→A) is another deontic axiom that seems desirable. Although it is wrong to say that if AA is obligatory then AA is the case (OA→A)(OA→A), still, this conditional ought to be the case. So some deontic logicians believe that DD needs to be supplemented with O(OA→A)O(OA→A) as well.

Controversy about iteration (repetition) of operators arises again in deontic logic. In some conceptions of obligation, OOAOOA just amounts to OAOA. 'It ought to be that it ought to be' is treated as a sort of stuttering; the extra 'ought's do not add anything new. So axioms are added to guarantee the equivalence of OOAOOA and OAOA. The more general iteration policy embodied in S5S5 may also be adopted. However, there are conceptions of obligation where distinction between OAOA and OOAOOA is preserved. The idea is that there are genuine differences between the obligations we actually have and the obligations we should adopt. So, for example, 'it ought to be that it ought to be that AA' commands adoption of some obligation which may not actually be in place, with the result that OOAOOA can be true even when OAOA is false.

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7.5 TEMPORAL LOGICS

In temporal logic (also known as tense logic), there are two basic operators, GG for the future, and HH for the past. GG is read 'it always will be that' and the defined operator FF (read 'it will be the case that') can be introduced by FA=∼G∼AFA=∼G∼A. Similarly HH is read: 'it always was that' and PP (for 'it was the case that') is defined by PA=∼H∼APA=∼H∼A. A basic system of temporal logic called KtKt results from adopting the principles of KK for both GG and HH, along with two axioms to govern the interaction between the past and future operators:

Necessitation Rules:

If AA is a theorem then so are GAGA and HAHA.

Distribution Axioms:

G(A→B)→(GA→GB)G(A→B)→(GA→GB) and H(A→B)→(HA→HB)H(A→B)→(HA→HB)

Interaction Axioms:

A→GPAA→GPA and A→HFAA→HFA

The interaction axioms raise questions concerning asymmetries between the past and the future. A standard intuition is that the past is fixed, while the future is still open. The first interaction axiom (A→GPA)(A→GPA) conforms to this intuition in reporting that what is the case (A)(A), will at all future times, be in the past (GPA)(GPA). However A→HFAA→HFA may appear to have unacceptably deterministic overtones, for it claims, apparently, that what is true now (A)(A) has always been such that it will occur in the future (HFA)(HFA). However, possible world semantics for temporal logic reveals that this worry results from a simple confusion, and that the two interaction axioms are equally acceptable.

Note that the characteristic axiom of modal logic, (M):ϒA→A(M):⌋A→A, is not acceptable for either HH or GG, since AA does not follow from 'it always was the case that AA', nor from 'it always will be the case that AA'. However, it is acceptable in a closely related temporal logic where GG is read 'it is and always will be', and HH is read 'it is and always was'.

Depending on which assumptions one makes about the structure of time, further axioms must be added to temporal logics. A list of axioms commonly adopted in temporal logics follows. An account of how they depend on the structure of time will be found in the section Possible Worlds Semantics.

GA→GGAGGA→GAGA→FA and HA→HHA and HHA→HA and HA →PAGA→GGA and HA→HHAGGA→GA and HHA→HAGA→FA a nd HA→PA

It is interesting to note that certain combinations of past tense and future tense operators may be used to express complex tenses in English. For example, FPAFPA, corresponds to sentence AA in the future perfect tense, (as in '20 seconds from now the light will have changed'). Similarly, PPAPPA expresses the past perfect tense.

7.6 CONDITIONAL LOGICS

The founder of modal logic, C. I. Lewis, defined a series of modal logics which did not have ϒ⌋ as a primitive symbol. Lewis was concerned to develop a logic of conditionals that was free of the so called Paradoxes of Material Implication, namely the classical theorems A→(∼A→B)A→(∼A→B) and B→(A→B)B→(A→B). He introduced the symbol ⥽⥽ for "strict implication" and developed logics where neither A⥽(∼A⥽B)A⥽(∼A⥽B) nor B⥽(A⥽B)B⥽(A⥽B) is provable. The modern practice has been to define A⥽BA⥽B by ϒ(A→B)⌋(A→B), and use modal logics governing ϒ⌋ to obtain similar results. However, the provability of such formulas as (A&∼A)⥽B(A&∼A)⥽B in such logics seems at odds with concern for the paradoxes. Anderson and Belnap (1975) have developed systems RR (for Relevance Logic) and EE (for Entailment) which are designed to overcome such difficulties. These systems require revision of the standard systems of propositional logic. (See Mares (2004) and the entry on relevance logic.)

David Lewis (1973) and others have developed conditional logics to handle counterfactual expressions, that is, expressions of the form 'if AA were to happen then BB would happen'. (Kvart (1980) is another good source on the topic.) Counterfactual logics differ from those based on strict implication because the former reject while the latter accept contraposition.

7.7 POSSIBLE WORLDS SEMANTICS

The purpose of logic is to characterize the difference between valid and invalid arguments. A logical system for a language is a set of axioms and rules designed to prove exactly the valid arguments statable in the language. Creating such logic may be a difficult task. The logician must make sure that the system is sound, i.e. that every argument proven using the rules and axioms is in fact valid. Furthermore, the system should be complete, meaning that every valid argument has a proof in the system. Demonstrating soundness and completeness of formal systems is a logician's central concern.

Such a demonstration cannot get underway until the concept of validity is defined rigorously. Formal semantics for a logic provides a definition of validity by characterizing the truth behavior of the sentences of the system. In propositional logic, validity can be defined using truth tables. A valid argument is simply one where every truth table row that makes its premises true also makes its conclusion true. However truth tables cannot be used to provide an account of validity in modal logics because there are no truth tables for expressions such as 'it is necessary that', 'it is obligatory that', and the like. (The problem is that the truth value of AA does not determine the truth value for ϒA⌋A. For example, when AA is 'Dogs are dogs', ϒA⌋A is true, but when AA is 'Dogs are pets', ϒA⌋A is false.) Nevertheless, semantics for modal logics can be defined by introducing possible worlds. We will illustrate possible worlds semantics for a logic of necessity containing the symbols ∼,→∼,→, and ϒ⌋. Then we will explain how the same strategy may be adapted to other logics in the modal family.

In propositional logic, a valuation of the atomic sentences (or row of a truth table) assigns a truth value (T(T or F)F) to each propositional variable pp. Then the truth values of the complex sentences are calculated with truth tables. In modal semantics, a set WW of possible worlds is introduced. A valuation then gives a truth value to each propositional variable for each of the possible worlds in WW. This means that value assigned to pp for world ww may differ from the value assigned to pp for another world w′w′.

The truth value of the atomic sentence pp at world ww given by the valuation vv may be written v(p,w)v(p,w). Given this notation, the truth values (T(T for true, FF for false) of complex sentences of modal logic for a given valuation vv (and member ww of the set of worlds W)W) may be defined by the following truth clauses. ('Iff' abbreviates 'if and only if'.)

v(∼A,w)=T iff v(A,w)=F.(∼)(∼)v(∼A,w)=T iff v(A,w)=F.

v(A→B,w)=T iff v(A,w)=F or v(B,w)=T.(→)(→)v(A→B,w)=T iff v(A, w)=F or v(B,w)=T.

v(□A,w)=T iff for every world w′ in W,v(A,w′)=T.(5)(5)v(⌋A,w)=T iff for every world w′ in W,v(A,w′)=T.

Clauses (∼)(∼) and (→)(→) simply describe the standard truth table behavior for negation and material implication respectively. According to (5), ϒA⌋A is true (at a world w)w) exactly when AA is true in all possible worlds. Given the definition of ◊◊, (namely, ◊A=∼ϒ∼A)◊A=∼⌋∼A) the truth condition (5) insures that ◊A◊A is true just in case AA is true in some possible world. Since the truth clauses for ϒ⌋ and ◊◊ involve the quantifiers 'all' and 'some' (respectively), the parallels in logical behavior between ϒ⌋ and ∀x∀x, and between ◊◊ and ∃x∃x noted in section 2 will be expected.

Clauses (∼),(→)(∼),(→), and (5) allow us to calculate the truth value of any sentence at any world on a given valuation. A definition of validity is now just around the corner. An argument is 5-valid for a given set W (of possible worlds) if and only if every valuation of the atomic sentences that assigns the premises TT at a world in WW also assigns the conclusion TT at the same world. An argument is said to be 5-valid iff it is valid for every non empty set WW of possible worlds.

It has been shown that S5S5 is sound and complete for 5-validity (hence our use of the symbol '5'). The 5-valid arguments are exactly the arguments provable in S5S5. This result suggests that S5S5 is the correct way to formulate logic of necessity.

However, S5S5 is not a reasonable logic for all members of the modal family. In deontic logic, temporal logic, and others, the analog of the truth condition (5) is clearly not appropriate; furthermore there are even conceptions of necessity where (5) should be rejected as well. The point is easiest to see in the case of temporal logic. Here, the members of WW are moments of time, or worlds “frozen”, as it were, at an instant. For simplicity let us consider a future temporal logic, a logic where ϒA⌋A reads: ‘it will always be the case that’. (We formulate the system using ϒ⌋ rather than the traditional GG so that the connections with other modal logics will be easier to appreciate.) The correct clause for ϒ⌋ should say that ϒA⌋A is true at time ww iff AA is true at all times in the future of ww. To restrict attention to the future, the relation RR (for ‘earlier than’) needs to be introduced. Then the correct clause can be formulated as follows.

v(ϒA,w)=T iff for every w′, if wRw′, then v(A,w′)=T.(K)(K)v(⌋A,w)=T iff for every w′, if wRw′, then v(A,w′)=T.

This says that ϒA⌋A is true at ww just in case AA is true at all times after ww.

Validity for this brand of temporal logic can now be defined. A frame ⟨W,R⟩⟨W,R⟩ is a pair consisting of a non-empty set WW (of worlds) and a binary relation RR on WW. A model ⟨F,v⟩⟨F,v⟩ consists of a frame FF, and a valuation vv that assigns truth values to each atomic sentence at each world in WW. Given a model, the values of all complex sentences can be determined using (∼),(→)(∼),(→), and (K)(K). An argument is KK-valid just in case any model whose valuation assigns the premises TT at a world also assigns the conclusion TT at the same world. As the reader may have guessed from our use of ‘KK’, it has been shown that the simplest modal logic KK is both sound and complete for KK-validity.

7.8 MODAL AXIOMS AND CONDITIONS ON FRAMES

One might assume from this discussion that KK is the correct logic when ϒ⌋ is read ‘it will always be the case that’. However, there are reasons for thinking that KK is too weak. One obvious logical feature of the relation RR (earlier than) is transitivity. If wRv(wwRv(w is earlier than v)v) and vRu(vvRu(v is earlier than u)u), then it follows that wRu(wwRu(w is earlier than u)u). So let us define a new kind of validity that corresponds to this condition on RR. Let a 4-model be any model whose frame ⟨W,R⟩⟨W,R⟩ is such that RR is a transitive relation on WW. Then an argument is 4-valid iff any 4-model whose valuation assigns TT to the premises at a world also assigns TT to the conclusion at the same world. We use '4' to describe such a transitive model because the logic which is adequate (both sounds and complete) for 4-validity is K4K4, the logic which results from adding the axiom (4): ϒA→ϒϒA⌋A→⌋⌋A to KK.

Transitivity is not the only property which we might want to require of the frame ⟨W,R⟩⟨W,R⟩ if RR is to be read 'earlier than' and WW is a set of moments. One condition (which is only mildly controversial) is that there is no last moment of time, i.e. that for every world ww there is some world vv such that wRvwRv. This condition on frames is called seriality. Seriality corresponds to the axiom (D):ϒA→◊A(D):⌋A→◊A, in the same way that transitivity corresponds to (4). A DD-model is a KK-model with a serial frame. From the concept of a DD-model the corresponding notion of DD-validity can be defined just as we did in the case of 4-validity. As you probably guessed, the system that is adequate with respect to DD-validity is KDKD, or KK plus (D)(D). Not only that, but the system KD4KD4 (that is KK plus (4) and (D))(D)) is adequate with respect to D4D4-validity, where a D4D4-model is one where ⟨W,R⟩⟨W,R⟩ is both serial and transitive.

Another property which we might want for the relation 'earlier than' is density, the condition which says that between any two times we can always find another. Density would be false if time were atomic, i.e. if there were intervals of time which could not be broken down into any smaller parts. Density corresponds to the axiom (C4):ϒϒA→ϒA(C4):⌋⌋A→⌋A, the converse of (4), so for example, the system KC4KC4, which is KK plus (C4)(C4) is adequate with respect to models where the frame ⟨W,R⟩⟨W,R⟩ is dense, and KDC4KDC4, adequate with respect to models whose frames are serial and dense, and so on.

Each of the modal logic axioms we have discussed corresponds to a condition on frames in the same way. The relationship between conditions on frames and corresponding axioms is one of the central topics in the study of modal logics. Once an interpretation of the intensional operator ϒ⌋ has been decided on, the appropriate conditions on RR can be determined to fix the corresponding notion of validity. This, in turn, allows us to select the right set of axioms for that logic.

For example, consider a deontic logic, where ϒ⌋ is read 'it is obligatory that'. Here the truth of ϒA⌋A does not demand the truth of AA in every possible world, but only in a subset of those worlds where people do what they ought. So we will want to introduce a relation RR for this kind of logic as well, and use the truth clause (K)(K) to evaluate ϒA⌋A at a world. However, in this case, RR is not earlier than. Instead wRw′wRw′ holds just in case world w′w′ is a morally acceptable variant of ww, i.e. a world that our actions can bring about which satisfies what is morally correct, or right, or just. Under such a reading, it should be clear that the relevant frames should obey seriality, the condition that requires that each possible world have a morally acceptable variant. The analysis of the properties desired for RR makes it clear that a basic deontic logic can be formulated by adding the axiom (D)(D) and to KK.

Even in modal logic, one may wish to restrict the range of possible worlds which are relevant in determining whether ϒA⌋A is true at a given world. For example, I might say that it is necessary for me to pay my bills, even though I know full well that there is a possible world where I fail to pay them. In ordinary speech, the claim that AA is necessary does not require the truth of AA in all possible worlds, but rather only in a certain class of worlds which I have in mind (for example, worlds where I avoid penalties for failure to pay). In order to provide a generic treatment of necessity, we must say that ϒA⌋A is true in ww iff AA is true in all worlds that are related to ww in the right way. So for an operator ϒ⌋ interpreted as necessity, we introduce a corresponding relation RR on the set of possible worlds WW, traditionally called the accessibility relation. The accessibility relation RR holds between worlds ww and w′w′ iff w′w′ is possible given the facts of ww. Under this reading for RR, it should be clear that frames for modal logic should be reflexive. It follows that modal logics should be founded on MM, the system that results from adding (M)(M) to KK. Depending on exactly how the accessibility relation is understood, symmetry and transitivity may also be desired.

A list of some of the more commonly discussed conditions on frames and their corresponding axioms along with a map showing the relationship between the various modal logics can be found in the next section.

7.9 MAP OF THE RELATIONSHIPS BETWEEN MODAL LOGICS

The following diagram shows the relationships between the best known modal logics, namely logics that can be formed by adding a selection of the axioms (D),(M)(D),(M), (4), (B)(B) and (5) to KK. A list of these (and other) axioms along with their corresponding frame conditions can be found below the diagram.

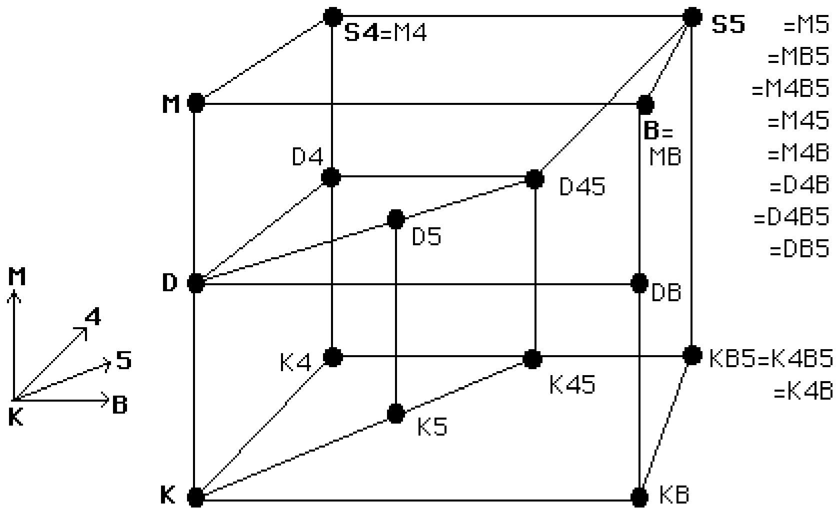

Diagram of Modal Logics

In this chart, systems are given by the list of their axioms. So, for example M4BM4B is the result of adding (M) (M), (4) and (B)(B) to KK. In boldface, we have indicated traditional names of some systems. When system SS appears below and/or to the left of S′S′ connected by a line, then S′S′ is an extension of SS. This means that every argument provable in SS is provable in S′S′, but SS is weaker than S′S′, i.e. not all arguments provable in S′S′ are provable in SS.

Notes

The following list indicates axioms, their names, and the corresponding conditions on the accessibility relation RR, for axioms so far discussed in this encyclopedia entry.

Name	**Axiom**	**Condition on Frames**	**R is…**
(D)(D)	ϒA→◊A⌋A→◊A	∃uwRu∃uwRu	Serial
(M)(M)	ϒA→A⌋A→A	wRwwRw	Reflexive
(4)	ϒA→ϒϒA⌋A→⌋⌋A	(wRv&vRu)⇒wRu(wRv&vRu)⇒wRu	Transitive
(B)(B)	A→ϒ◊AA→⌋◊A	wRv⇒vRwwRv⇒vRw	Symmetric
(5)	◊A→ϒ◊A◊A→⌋◊A	(wRv&wRu)⇒vRu(wRv&wRu)⇒vRu	Euclidean
(CD)(CD)	◊A→ϒA◊A→⌋A	(wRv&wRu)⇒v=u(wRv&wRu)⇒v=u	Functional
(ϒM)(⌋M)	ϒ(ϒA→A)⌋(⌋A→A)	wRv⇒vRvwRv⇒vRv	Shift Reflexive
(C4)(C4)	ϒϒA→ϒA⌋⌋A→⌋A	wRv⇒∃u(wRu&uRv)wRv⇒∃u(wRu&uRv)	Dense
(C)(C)	◊ϒA→ϒ◊A◊⌋A→⌋◊A	wRv&wRx⇒∃u(vRu&xRu)wRv&wRx⇒∃u(vRu&xRu)	Convergent

In the list of conditions on frames, and in the rest of this article, the variables 'ww', 'vv', 'uu', 'xx' and the quantifier '∃u∃u' are understood to range over WW. '&' abbreviates 'and' and '⇒⇒' abbreviates 'if…then'.

The notion of correspondence between axioms and frame conditions that is at issue here was explained in the previous section. When S is a list of axioms and F(S) is the corresponding set of frame conditions, then S corresponds to F(S) exactly when the system K+S is adequate (sound and complete) for F(S)-validity, that is, an argument is provable in K+S iff it is F(S)-valid. Several stronger notions of correspondence between axioms and frame conditions have emerged in research on modal logic.

7.10 THE GENERAL AXIOM

The correspondence between axioms and conditions on frames may seem something of a mystery. A beautiful result of Lemmon and Scott (1977) goes a long way towards explaining those relationships. Their theorem concerned axioms which have the following form:

◊hϒiA→ϒj◊kA(G)(G)◊h⌋iA→⌋j◊kA

We use the notation '◊n◊n' to represent nn diamonds in a row, so, for example, '◊3◊3' abbreviates a string of three diamonds: '◊◊◊◊◊◊'. Similarly 'ϒn⌋n' represents a string of nn boxes. When the values of h,i,jh,i,j, and kk are all 1, we have axiom (C)(C):

◊ϒA→ϒ◊A=◊1ϒ1A→ϒ1◊1A(C)(C)◊⌋A→⌋◊A=◊1⌋1A→⌋1◊1A

The axiom (B)(B) results from setting hh and ii to 0, and letting jj and kk be 1:

A→ϒ◊A=◊0ϒ0A→ϒ1◊1A(B)(B)A→⌋◊A=◊0⌋0A→⌋1◊1A

To obtain (4), we may set hh and kk to 0, set ii to 1 and jj to 2:

ϒA→ϒϒA=◊0ϒ1A→ϒ2◊0A(4)(4)⌋A→⌋⌋A=◊0⌋1A→⌋2◊0A

Many (but not all) axioms of modal logic can be obtained by setting the right values for the parameters in (G)(G)

Our next task will be to give the condition on frames which corresponds to (G)(G) for a given selection of values for h,i,jh,i,j, and kk. In order to do so, we will need a definition. The composition of two relations RR and R′R′ is a new relation R∘R′R∘R′ which is defined as follows:

wR∘R′v iff for some u,wRu and uR′v.wR∘R′v iff for some u,wRu and uR′v.

For example, if RR is the relation of being a brother, and R′R′ is the relation of being a parent then R∘R′R∘R′ is the relation of being an uncle, (because ww is the uncle of vv iff for some person uu, both ww is the brother of uu and uu is the parent of v)v). A relation may be composed with itself. For example, when RR is the relation of being a parent, then R∘RR∘R is the relation of being a grandparent, and R∘R∘RR∘R∘R is the relation of being a great-grandparent. It will be useful to write 'RnRn', for the result of composing RR with itself nn times. So R2R2 is R∘RR∘R, and R4R4 is R∘R∘R∘RR∘R∘R∘R. We will let R1R1 be RR, and R0R0 will be the identity relation, i.e. wR0vwR0v iff w=vw=v.

We may now state the Scott-Lemmon result. It is that the condition on frames which corresponds exactly to any axiom of the shape (G)(G) is the following.

wRhv&wRju⇒∃x(vRix&uRkx)(hijk-Convergence)(hijk-Convergence)wRhv&wRju⇒∃x(vRix&uRkx)

It is interesting to see how the familiar conditions on RR result from setting the values for hh, ii, jj, and kk according to the values in the corresponding axiom. For example, consider (5). In this case i=0i=0, and h=j=k=1h=j=k=1. So the corresponding condition is

wRv&wRu⇒∃x(vR0x&uRx).wRv&wRu⇒∃x(vR0x&uRx).

We have explained that R0R0 is the identity relation. So if vR0xvR0x then v=xv=x. But ∃x(v=x&uRx)∃x(v=x&uRx), is equivalent to uRvuRv, and so the Euclidean condition is obtained:

(wRv&wRu)⇒uRv.(wRv&wRu)⇒uRv.

In the case of axiom (4), h=0,i=1,j=2h=0,i=1,j=2 and k=0k=0. So the corresponding condition on frames is

(w=v&wR2u)⇒∃x(vRx&u=x).(w=v&wR2u)⇒∃x(vRx&u=x).

Resolving the identities this amounts to:

vR2u⇒vRu.vR2u⇒vRu.

By the definition of R2,vR2uR2,vR2u iff ∃x(vRx&xRu)∃x(vRx&xRu), so this comes to:

∃x(vRx&xRu)⇒vRu,∃x(vRx&xRu)⇒vRu,

which by predicate logic, is equivalent to transitivity.

vRx&xRu⇒vRu.vRx&xRu⇒vRu.

The reader may find it a pleasant exercise to see how the corresponding conditions fall out of hijk-Convergence when the values of the parameters hh, ii, jj, and kk are set by other axioms.

The Scott-Lemmon results provides a quick method for establishing results about the relationship between axioms and their corresponding frame conditions. Since they showed the adequacy of any logic that extends KK with a selection of axioms of the form (G)(G) with respect to models that satisfy the corresponding set of frame conditions, they provided "wholesale" adequacy proofs for the majority of systems in the modal family. Sahlqvist (1975) has discovered important generalizations of the Scott-Lemmon result covering a much wider range of axiom types. The reader should be warned, however, that the neat correspondence between axioms and conditions on frames is atypical. There are condtions on frames that correspond to no axioms, and there are even conditions on frames for which no system is adequate. (For an example see Boolos, 1993, pp. 148ff.)

7.11 TWO DIMENSIONAL SEMANTICS

Two dimensional semantics is a variant of possible world semantics that uses two (or more) kinds of parameters in truth evaluation, rather than possible worlds alone. For example, a logic of indexical expressions, such as ‘I’, ‘here’, ‘now’, and the like, needs to bring in the linguistic context (or context for short). Given a context c=⟨s,p,t⟩c=⟨s,p,t⟩ where ss is the speaker, pp the place, and tt the time of utterance, then ‘I’ refers to ss, ‘here’ to pp, and ‘now’ to tt. So in the context c=⟨c=⟨Jim Garson, Houston, 3:00 P.M. CST on 4/3/2014⟩2014⟩ ‘I am here now’ is T iff Jim Garson is in Houston, at 3:00 P.M. CST on 4/3/2014.

In possible worlds semantics, a sentence’s truth-value depended on the world at which it is evaluated. However, indexicals bring in a second dimension – so we need to generalize again. Kaplan (1989) defines the character of a sentence B to be a function from the set of (linguistic) contexts to the content (or intension) of B, where the content, in turn, is simply the intension of B, that is a function from possible worlds to truth-values. Here, truth evaluation is doubly dependent – on both linguistic contexts and possible worlds.

One of Kaplan’s most interesting observations is that some indexical sentences are contingent, but at the same time analytically true. An example is (1).

- (1)I am here now.

Just from the meaning of the words, you can see that (1) must be true in any context c=⟨s,p,t⟩c=⟨s,p,t⟩. After all, cc counts as a linguistic context just in case ss is a speaker who is at place pp at time tt. Therefore (1) is true at cc, and that means that the pattern of truth-values (1) has along the context dimension must be all Ts (given the possible world is held fixed). This suggests that the context dimension is apt for tracking analytic knowledge obtained from the mastery of our language. On the other hand, the possible-worlds dimension keeps track of what is necessary. Holding the context fixed, there there are possible worlds where (1) is false. For example, when c=⟨c=⟨Jim Garson, Houston, 3:00 P.M. CST on 4/3/2014⟩2014⟩, (1) fails at cc in a possible world where Jim Garson is in Boston at 3:00 P.M. CST on 4/3/2014. It follows that ‘I am here now’ is a contingent analytic truth. Therefore, two-dimensional semantics can handle situations where necessity and analyticity come apart.

Another example where bringing in two dimension is useful is in the logic for an open future (Thomason, 1984; Belnap, et al., 2001). Here one employs a temporal structure where many possible future histories extend from a given time. Consider (2).

- (2)Joe will order a sea battle tomorrow.

If (2) is contingent, then there is a possible history where the battle occurs the day after the time of evaluation, and another one where it does not occur then. So to evaluate (2) you need to know two things: what is the time t of evaluation, and which of the histories h that run through t is the one to be considered. So a sentence in such a logic is evaluated at a pair ⟨t,h⟩⟨t,h⟩.

Another problem resolved by two-dimensional semantics is the interaction between ‘now’ and other temporal expressions like the future tense ‘it will be the case that’. Then it is plausible to think that ‘now’ refers to the time of evaluation. So we would have the following truth condition:

v(NowB,t)=T iff v(B,t)=T.(Now)(Now)v(NowB,t)=T iff v(B,t)=T.

However this will not work for sentences like (3).

- (3)At some point in the future, everyone now living will be unknown.

With FF as the future tense operator, (3) might be translated:

F∀x(NowLx→Ux).(3′)(3′)F∀x(NowLx→Ux).

(The correct translation cannot be ∀x(NowLx→FUx)∀x(NowLx→FUx), with FF taking narrow scope, because (3) says there is a future time when all things now living are unknown together, not that each living thing will be unknown in some future time of its own). When the truth conditions for (3)″ calculated, using (Now) and the truth condition (FF) for FF, it turns out that (3)″ is true at time uu iff there is a time tt after uu such that everything that is living at tt (not uu!) is unknown at tt.

v(FB,t)=T iff for some time u later than t,v(B,u)=T.(F)(F)v(FB,t)=T iff for some time u later than t,v(B,u)=T.

To evaluate (3)″ correctly so that it matches what we mean by (3), we must make sure that 'now' always refers back to the original time of utterance when 'now' lies in the scope of other temporal operators such as F. Therefore we need to keep track of which time is the time of utterance (u)(u) as well as which time is the time of evaluation (t)(t). So our indices take the form of a pair ⟨u,e⟩⟨u,e⟩, where uu is the time of utterance, and ee is the time of evaluation. Then the truth condition (Now) is revised to (2DNow).

v(NowB,⟨u,e⟩)=T iff v(B,⟨u,u⟩)=T.(2DNow)(2DNow)v(NowB,⟨u,e⟩)=T iff v(B,⟨u,u⟩)=T.

This has it that the NowBB is true at a time u of utterance and time e of evaluation provided that B is true when u is taken to be the time of evaluation. When the truth conditions for F, ∀∀, and →→ are revised in the obvious way (just ignore the u in the pair), (3)″ is true at ⟨u,e⟩⟨u,e⟩ provided that there is a time e′e′ later than e such that everything that is living at uu is unknown at e′e′. By carrying along a record of what uu is during the truth calculation, we can always fix the value for 'now' to the original time of utterance, even when 'now' is deeply embedded in other temporal operators.

A similar phenomenon arises in modal logics with an actuality operator A (read 'it is actually the case that'). To properly evaluate (4) we need to keep track of which world is taken to be the actual (or real) world as well as which one is taken to the world of evaluation.

- (4)It is possible that everyone actually living be unknown.

The idea of distinguishing different possible world dimensions in semantics has had useful applications in philosophy. For example, Chalmers (1996) has presented arguments from the conceivability of (say) zombies to dualist conclusions in the philosophy of mind. Chalmers

(2006) has deployed two-dimensional semantics to help identify an a priori aspect of meaning that would support such conclusions.

The idea has also been deployed in the philosophy of language. Kripke (1980) famously argued that 'Water is H2O' is a posteriori but nevertheless a necessary truth, for given that water just is H20, the there is no possible world where THAT stuff is (say) a basic element as the Greeks thought. On the other hand, there is a strong intuition that had the real world been somewhat different from what it is, the odorless liquid that falls from the sky as rain, fills our lakes and rivers, etc. might perfectly well have been an element. So in some sense it is conceivable that water is not H20. Two dimensional semantics makes room for these intuitions by providing a separate dimension that tracks a conception of water that lays aside the chemical nature of what water actually is. Such a 'narrow content' account of the meaning of 'water' can explain how one may display semantical competence in the use of that term and still be ignorant about the chemistry of water (Chalmers, 2002).

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What is Modal Logic?

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2. What is Modal Logics?

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3. Write about Deontic Logics.

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7.12 PROVABILITY LOGICS

Modal logic has been useful in clarifying our understanding of central results concerning provability in the foundations of mathematics (Boolos, 1993). Provability logics are systems where the propositional variables p,q,rp,q,r, etc. range over formulas of some mathematical system, for example Peano's system PAPA for arithmetic. (The system chosen for mathematics might vary, but assume it is PAPA for this discussion.) Gödel showed that arithmetic has strong expressive powers. Using code numbers for arithmetic sentences, he was able to demonstrate a correspondence between sentences of mathematics and facts about which sentences are and are not provable in PAPA. For example, he showed there there is a sentence CC that is true just in case no contradiction is provable in PAPA and there is a sentence GG (the famous Gödel sentence) that is true just in case it is not provable in PAPA.

In provability logics, ϒp⌋p is interpreted as a formula (of arithmetic) that expresses that what pp denotes is provable in PAPA. Using this notation, sentences of provability logic express facts about provability. Suppose that ⊥⊥ is a constant of provability logic denoting a contradiction. Then ∼ϒ⊥∼⌋⊥ says that PAPA is consistent and ϒA→A⌋A→A says that PAPA is sound in the sense that when it proves A,AA,A is indeed true. Furthermore, the box may be iterated. So, for example, ϒ∼ϒ⊥⌋∼⌋⊥ makes the dubious claim that PAPA is able to prove its own consistency, and ∼ϒ⊥→∼ϒ∼ϒ⊥∼⌋⊥→∼⌋∼⌋⊥ asserts (correctly as Gödel proved) that if PAPA is consistent then PAPA is unable to prove its own consistency.

Although provability logics form a family of related systems, the system GLGL is by far the best known. It results from adding the following axiom to KK:

ϒ(ϒA→A)→ϒA(GL)(GL)⌋(⌋A→A)→⌋A

The axiom (4): ϒA→ϒϒA⌋A→⌋⌋A is provable in GLGL, so GLGL is actually a strengthening of K4K4. However, axioms such as (M):ϒA→A(M):⌋A→A, and even the weaker (D):ϒA→◊A(D):⌋A→◊A are not available (nor desirable) in GLGL. In provability logic, provability is not to be treated as a brand of necessity. The reason is that when pp is provable in an arbitrary system SS for mathematics, it does not follow that pp is true, since SS may be unsound. Furthermore, if pp is provable in S(ϒp)S(⅃p) it need not even follow that ∼p∼p lacks a proof (∼ϒ∼p=◊p).S(∼⅃∼p=◊p).S might be inconsistent and so prove both pp and ∼p∼p.

Axiom (GL)(GL) captures the content of Loeb's Theorem, an important result in the foundations of arithmetic. □A→A☐A→A says that PAPA is sound for AA, i.e. that if AA were proven, A would be true. (Such a claim might not be secure for an arbitrarily selected system SS, since A might be provable in SS and false.) (GL)(GL) claims that if PAPA manages to prove the sentence that claims soundness for a given sentence AA, then AA is already provable in PAPA. Loeb's Theorem reports a kind of modesty on PAPA's part (Boolos, 1993, p. 55). PAPA never insists (proves) that a proof of AA entails AA's truth, unless it already has a proof of AA to back up that claim.

It has been shown that GLGL is adequate for provability in the following sense. Let a sentence of GLGL be always provable exactly when the sentence of arithmetic it denotes is provable no matter how its variables are assigned values to sentences of PAPA. Then the provable sentences of GLGL are exactly the sentences that are always provable. This adequacy result has been extremely useful, since general questions concerning provability in PAPA can be transformed into easier questions about what can be demonstrated in GLGL.

GLGL can also be outfitted with a possible world semantics for which it is sound and complete. A corresponding condition on frames for GLGL-validity is that the frame be transitive, finite and irreflexive.

7.13 ADVANCED MODAL LOGIC

The applications of modal logic to mathematics and computer science have become increasingly important. Provability logic is only one example of this trend. The term "advanced modal logic" refers to a tradition in modal logic research that is particularly well represented in departments of mathematics and computer science. This tradition has been woven into the history of modal logic right from its beginnings (Goldblatt, 2006). Research into relationships with topology and algebras represents some of the very first technical work on modal logic. However the term 'advanced modal logic' generally refers to a second wave of work done since the mid 1970s. Some examples of the many interesting topics dealt with include results on decidability (whether it is possible to compute whether a formula of a given modal logic is a theorem) and complexity (the costs in time and memory needed to compute such facts about modal logics).

7.14 BISIMULATION

Bisimulation provides a good example of the fruitful interactions that have been developed between modal logic and computer science. In computer science, labeled transition systems (LTSs) are commonly used to represent possible computation pathways during execution of a program. LTSs are generalizations of Kripke frames, consisting of a set W of states, and a collection of i-accessibility relations Ri, one for each computer process i. Intuitively, wRiw′ holds exactly when w′ is a state that results from applying the process i to state w.

The language of poly-modal or dynamic logic introduces a collection of modal operators ϒi, one for each program i (Harel, 1984). Then ϒiA states that sentence A holds in every result of applying i. So ideas like the correctness and successful termination of programs can be expressed in this language. Models for such a language are like Kripke models save that LTSs are used in place of frames. A bisimulation is a counterpart relation between states of two such models such that exactly the same propositional variables are true in counterpart states, and whenever world v is i-accessible from one of two counterpart states, then the other counterpart bears the i-accessibility relation to some counterpart of v. In short, the i-accessibility structure one can "see" from a given state mimics what one sees from a counterpart. Bisimulation is a weaker notion than isomorphism (a bisimulation relation need not be 1-1), but it is sufficient to guarantee equivalence in processing.

In the 1970s, a version of bisimulation had already been developed by modal logicians to help better understand the relationship between modal logic axioms and their corresponding conditions on Kripke frames. Kripke's semantics provides a basis for translating modal axioms into sentences of a second-order language where quantification is allowed over one-place predicate letters P. Replace metavariables A with open sentences Px, translate ΥPx to $\forall y(Rxy \rightarrow Py)$, and close free variables x and predicate letters P with universal quantifiers. For example, the predicate logic translation of the axiom schema $\Upsilon A \rightarrow A$ comes to $\forall P \forall x[\forall y(Rxy \rightarrow Py) \rightarrow Px]$. Given this translation, one may instantiate the variable P to an arbitrary one-place predicate, for example to the predicate Rx whose extension is the set of all worlds w such that Rxw for a given value of x. Then one obtains $\forall x[\forall y(Rxy \rightarrow Rxy) \rightarrow Rxx]$, which reduces to $\forall xRxx$, since $\forall y(Rxy \rightarrow Rxy)$ is a tautology. This illuminates the correspondence between $\Upsilon A \rightarrow A$ and reflexivity of frames ($\forall xRxx$). Similar results hold for many other axioms and frame conditions. The "collapse" of second-order axiom conditions to first order frame conditions is very helpful in obtaining completeness results for modal logics. For example, this is the core idea behind the elegant results of Sahlqvist (1975).

But when does the second-order translation of an axiom reduce to a first-order condition on R in this way? In the 1970s, van Benthem showed that this happens iff the translation's holding in a model entails its holding in any bisimular model, where two models are bisimular iff there is a bisimulation between them in the special case where there is a single accessibility relation. That result generalizes easily to the poly-modal case (Blackburn et. al., 2001, p. 103). This suggests that poly-modal logic lies at exactly the right level of abstraction to describe, and reason about, computation and other processes. (After all, what really matters there is the preservation of truth values of formulas in models rather than the finer details of the frame structures.) Furthermore the implicit translation of those logics into well-understood fragments of predicate logic provides a wealth of information of interest to computer scientists. As a result, a fruitful area of research in computer science has developed with bisimulation as its core idea (Ponse et al. 1995).

7.15 MODAL LOGIC AND GAMES

The interaction between the theory of games and modal logic is a flourishing new area of research (van der Hoek and Pauly, 2007; van

Benthem, 2011, Ch. 10, and 2014). This work has interesting applications to understanding cooperation and competition among agents as information available to them evolves.

The Prisoner's Dilemma illustrates some of the concepts in game theory that can be analyzed using modal logics. Imagine two players that choose to either cooperate or cheat. If both cooperate, they both achieve a reward of 3 points, if they both cheat, they both get nothing, and if one cooperates and the other cheats, the cheater makes off with 5 points and the cooperator gets nothing. If both players are altruistic and motivated to maximize the sum of their rewards, they will both cooperate, as this is the best they can do together. However, they are both tempted to cheat to increase their own reward from 3 to 5. On the other hand, if they are rational, they may recognize that if they cheat their opponent threatens to cheat and leave them with nothing. So cooperation is the best one can do given this threat. And if each thinks the other realizes this, they may be motivated to cooperate. An extended (or iterated) version of this game gives the players multiple moves, that is, repeated opportunities to play and collect rewards. If players have information about the history of the moves and their outcomes, new concerns come into play, as success in the game depends on knowing their opponent's strategy, and determining (for example) when he/she can be trusted not to cheat. In multi-player versions of the game, where players are drawn in pairs from a larger pool at each move, one's own best strategy may well depend on whether one can recognize one's opponents and the strategies they have adopted. (See Grim et. al., 1998 for fascinating research on Interated Prisoner's Dilemmas.)

In games like Chess, players take turns making their moves and their opponents can see the moves made. If we adopt the convention that the players in a game take turns making their moves, then the Iterated Prisoner's Dilemma is a game with missing information about the state of play – the player with the second turn lacks information about what the other player's last move was. This illustrates the interest of games with imperfect information.

The application of games to logic has a long history. One influential application with important implications for linguistics is Game Theoretic

Semantics (GTS) (Hintikka et. al. 1983), where validity is defined by the outcome of a game between two players one trying to verify and the other trying to falsify a given formula. GTS has significantly stronger resources that standard Tarski-style semantics, as it can be used (for example) to explain how meaning evolves in a discourse (a sequence of sentences).

However, the work on games and modal logic to be described here is somewhat different. Instead of using games to analyze the semantics of a logic, the modal logics at issue are used to analyze games. The structure of games and their play is very rich, as it involves the nature of the game itself (the allowed moves, and the rewards for the outcomes), the strategies (which are sequences of moves through time), and the flow of information available to the players as the game progresses. Therefore, the development of modal logic for games draws on features found in logics involving concepts like time, agency, preference, goals, knowledge, belief, and cooperation.

To provide some hint at this variety, here is a limited description of some of the modal operators that turn up in the analysis of games and some of the things that can be expressed with them. The basic idea in the semantics is that a game consists of a set of players 1, 2, 3, …, and a set of W of game states. For each player i, there is an accessibility relation RiRi understood so that sRitsRit holds for states ss and tt iff when the game has come to state ss player ii has the option of making a move that results in tt. This collection of relations defines a tree whose branches define every possible sequence of moves in the game. The semantics also assigns truth-values to atoms that keep track of the payoffs. So, for example in a game like Chess, there could be an atom winiwini such that v(wini,s)=Tv(wini,s)=T iff state s is a win for player ii. Model operators ϒi⌋i and ◊i◊i for each player i may then be defined as follows.

v(ϒiA,s)v(◊iA,s)=T iff for all t in W, if sRit, then v(A,t)=T.=T iff for some t in W,sRit and v(A,t)=T.v(⌋iA,s)=T iff for all t in W, if sRit, then v(A,t)=T.v(◊iA,s)=T iff for some t in W,sRit and v(A,t)=T.

So ϒiA⌋iA (◊iA)(◊iA) is true in s provided that sentence AA holds true in every (some) state that ii can chose from state ss. Given that ⊥⊥ is a contradiction (so ~⊥~⊥ is a tautology), ◊i~⊥◊i~⊥ is true at a state when it is ii's turn to move. For a two-player game ϒ1⊥⌋1⊥ & ϒ2⊥⌋2⊥ is true of a state that ends the game, because neither 1 nor 2 can move.□ϒ1◊2⌋1◊2win22 asserts that player 1 has a loss because whatever 1 does from the present state, 2 can win in the following move.

For a more general account of the player's payoffs, ordering relations ≤i≤i can be defined over the states so that s≤its≤it means that ii's payoff for tt is at least as good as that for ss. Another generalization is to express facts about sequences qq of moves, by introducing operators interpreted by relations sRqtsRqt indicating that the sequence qq starting from s eventually arrives at tt. With these and related resources, it is possible express (for example) that q is ii's best strategy given the present state.

It is crucial to the analysis of games to have a way to express the information available to the players. One way to accomplish this is to borrow ideas from epistemic logic. Here we may introduce an accessibility relation ~i~i for each player such that s~its~it holds iff ii cannot distinguish between states ss and tt. Then knowledge operators KiKi for the players can be defined so that KiAKiA says at ss that AA holds in all worlds that ii can distinguish from ss; that is, despite ii's ignorance about the state of play, he/she can still be confident that AA. KK operators may be used to say that player 1 is in a position to resign, for he knows that 2 sees she has a win: K1K2ϒ1◊2win2K1K2⌋1◊2win2.

Since player's information varies as the game progresses, it is useful to think of moves of the game as indexed by times, and to introduce operators OO and UU from tense logic for 'next' and 'until'. Then KiOA→OKiAKiOA→OKiA expresses that player ii has "perfect recall", that is, that when ii knows that AA happens next, then at the next moment ii has not forgotten that AA has happened. This illustrates how modal logics for games can reflect cognitive idealizations, and a player's success (or failure) at living up to them.

The technical side of the modal logics for games is challenging. The project of identifying systems of rules that are sound and complete for a language containing a large collection of operators may be guided by past research, but the interactions between the varieties of accessibility relations leads to new concerns. Furthermore, the computational complexity of various systems and their fragments is a large landscape largely unexplored.

Game theoretic concepts can be applied in a surprising variety of ways – from checking an argument for validity to succeeding in the political arena. So there are strong motivations for formulating logics that can handle games. What is striking about this research is the power one obtains by weaving together logics of time, agency, knowledge, belief, and preference in a unified setting. The lessons learned from that integration have value well beyond what they contribute to understanding games.

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Discuss about Temporal Logics

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2. What is Conditional Logics?

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3. Discuss the Possible Worlds Semantics.

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4. What is Modal Axioms and Conditions on Frame?

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7.16 LET US SUM UP

A final complication in the semantics for quantified modal logic is worth mentioning. It arises when non-rigid expressions such as ‘the inventor of bifocals’ are introduced to the language. A term is non-rigid when it picks out different objects in different possible worlds. The semantical value of such a term can be given by what Carnap (1947) called an individual concept, a function that picks out the denotation of the term for each possible world. One approach to dealing with non-rigid terms is to employ Russell’s theory of descriptions. However, in a language that treats non rigid expressions as genuine terms, it turns out that neither the classical nor the free logic rules for the quantifiers are acceptable. (The problem can not be resolved by weakening the rule of substitution for identity.) A solution to this problem is to employ a more general treatment of the quantifiers, where the domain of quantification contains individual concepts rather than objects. This more general interpretation provides a better match between the treatment of terms and the treatment of quantifiers and results in systems that are adequate for classical or free logic rules (depending on whether the fixed domains or world-relative domains are chosen). It also provides a language with strong and much needed expressive powers (Bressan, 1973, Belnap and Müller, 2013a, 2013b).

7.17 KEY WORDS

Semantics: Semantics is the linguistic and philosophical study of meaning in language, programming languages, formal logics, and semiotics. It is concerned with the relationship between signifiers—like words, phrases, signs, and symbols—and what they stand for in reality, their denotation.

7.18 QUESTIONS FOR REVIEW

Notes

1. What is the Map of the Relationships Between Modal Logics?
2. What is The General Axiom?
3. Discuss about two Dimensional Semantics
4. What is Provability Logics?
5. Discuss Advanced Modal Logic
6. What is Bisimulation?
7. What is Modal Logic and Games?

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7.20 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 7.2
2. See Section 7.3
3. See Section 7.4

Check Your Progress 2

1. See Section 7.5
2. See Section 7.6
3. See Section 7.7
4. See Section 7.8